Master's Thesis Topic:
Parameterized Complexity – Structural and Fine-Grained
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Project Description

Keywords:

- Graphs and Structural Graph Parameters (treewidth, pathwidth, ...)
- Computational Complexity, Parameterized Complexity
- Fine-Grained Complexity, ETH, SETH

Short summary: We will work on optimization problems on **graphs** which are known to be NP-hard, such as (generalizations of) Independent Set and Dominating Set. These problems require exponential time to solve exactly! Therefore, we will concentrate on structured graphs. Our goal is to understand how the complexity of such problems depends not on the input size, but on the input **structure**. For this, we will need to formulate both algorithms and reductions (hopefully) proving that our algorithms are optimal.

Scientific Context: The question of dealing with the phenomenon of NP-hardness is of central importance to the field of theoretical computer science, because NP-hard problems abound in all fields of science and engineering. Unfortunately, such problem are widely believed to be intractable (under standard assumptions such as P≠NP). In this project we focus on **parameterized complexity** approaches for dealing with NP-hard problems, with an emphasis on **structural parameters** and **fine-grained complexity**.

In a few words, parameterized complexity studies algorithms which have running times which are exponential in the worst case, but where the exponential function can be confined in a parameter that measures the *structure* rather than the size of the input, hence this field aims to use measures of input structure to tame NP-hardness [1]. We will mainly deal with graph problems, so the measures of input structure we mostly care about will be structural graph parameters, such as **treewidth**. In this context we will ask questions of the following form: "We know that problem Π requires time exponential in n to solve exactly, but how does its complexity depend on the structure of the input graph?". In particular, we care about whether NP-hard problems can be solved in time $f(k)n^{O(1)}$, where k is a measure of structure. Furthermore, beyond this qualitative classification, we will aim to precisely pinpoint the best possible function $f(k)$ for which such a running time is possible. This is part of **fine-grained complexity** theory, a more refined version of computational complexity, which rather than P≠NP and related assumptions, relies on conjectures such as the (Strong) Exponential Time Hypothesis (ETH and SETH [3]).

Project Goals: The intended research goal is to produce new fine-grained tight bounds on the complexity of interesting optimization problems on graphs. Along the way, you will gain experience on (i) some of the main techniques of parameterized algorithm design, such as DP algorithms on tree decompositions (ii) fine-grained tools in parameterized complexity, such as reductions based on the ETH and/or SETH.

To get a feeling for what we are looking for, some results in the same spirit can be found in [2, 4, 5, 6]. Observe that the message of these works is to pinpoint that for several famous problems (Coloring, Dominating Set, Hamiltonicity...) the **correct** complexity is $c^k n^{O(1)}$, where k is the input graph's treewidth/pathwidth. It is important to note that we know algorithms and lower bounds that *match* and that the constant c is specified exactly, often with unexpected values. To see why this research is exciting, ask yourselves how often in theoretical computer science we are able to calculate an interesting constant to its exact value...

Practical Information

The proposed work will be carried out under the supervision of Michael Lampis, hosted in LAMSADE, Université Paris-Dauphine. LAMSADE, a CNRS lab attached to Dauphine, is a world-class research center in theoretical computer science, with a vibrant and international research environment. Université Paris-Dauphine is one of the leading universities in Paris, located in the 16th district, a few minutes away from the Arc de Triomphe.

There is a possibility of funding for this master's internship via the ANR-funded project Sub-EXponential APproximation and ParametErized ALgorithms (S-EX-AP-PE-AL)[1]. Furthermore, there is a possibility to continue along this line of research with the goal of obtaining a doctorate.

Candidate Profile

In order to apply you should already be (at least) familiar with:

- The basics of algorithms analysis and computational complexity theory (e.g. P vs. NP).
- Basics of graph theory and graph algorithms.
- Previous knowledge of parameterized complexity is not necessary but a plus.

In other words, you should consider applying for this project if (i) you like cool algorithmic problems on graphs (ii) you enjoyed doing reductions to prove NP-completeness (iii) you want to participate in a modern line of research extending these questions.
Interested candidates are invited to send informal inquiries to Michael Lampis `michail.lampis@dauphine.fr` before submitting a full application.

References

[1] Marek Cygan, Fedor V. Fomin, Lukasz Kowalik, Daniel Lokshtanov, Dániel Marx, Marcin Pilipczuk, Michal Pilipczuk, and Saket Saurabh. *Parameterized Algorithms*. Springer, 2015.

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[4] Michael Lampis. Finer tight bounds for coloring on clique-width. *SIAM J. Discret. Math.*, 34(3):1538–1558, 2020.

[5] Michael Lampis and Manolis Vasilakis. Structural parameterizations for two bounded degree problems revisited. In *31st Annual European Symposium on Algorithms, ESA 2023*, volume 274 of *LIPIcs*, pages 77:1–77:16. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2023.

[6] Daniel Lokshtanov, Dániel Marx, and Saket Saurabh. Known algorithms on graphs of bounded treewidth are probably optimal. *ACM Trans. Algorithms*, 14(2):13:1–13:30, 2018.

[1] `https://www.lamsade.dauphine.fr/~mlampis/SEXAPPEAL`