TD 3: Bipartite Graphs

1 Turan

What is the maximum number of edges of a bipartite graph with n vertices?

2 Regularity Makes Perfect

Recall that a graph is k-regular if all vertices have degree exactly k. Show that for all $k > 0$, if a bipartite graph is k-regular, then it has a perfect matching. (Note: first convince yourselves that if a bipartite graph is k-regular, then its two parts have the same size.)

3 Maximal Matchings

A *maximal* matching M is a matching such that $M + e$, where e is an edge not in M, is no longer a matching. Prove that if M is a maximal matching and M' is a maximum matching, then $|M|$ is at least $|M'|/2$.

4 Dominating Set

We saw in class that the MINIMUM VERTEX COVER problem is easier on bipartite graphs than it is on general graphs. For this exercise we look at a problem which is as hard on general graphs as it is on bipartite graphs. Recall that a *dominating set* of a graph $G = (V, E)$ is a set $S \subseteq V$ such that all vertices of $V \setminus S$ have a neighbor in S. In the MINIMUM DOMINATING SET problem we are given G, k and are asked if G has a dominating set of size at most k. Show that if we had an efficient algorithm for MINIMUM DOMINATING SET on bipartite graphs, we would have such an algorithm for the same problem on general graphs. (Hint: Given an arbitrary graph G you must modify it so that you construct a bipartite graph G' but preserve the solution.)

5 Perfect Matchings on Trees

Show that a tree has a perfect matching if and only if for all v, o(G-v)=1, where o(G) is the number of odd-order components.