TD 12: Revision

1 Outerplanar Graphs and Kuratowski

Show that a graph G is outerplanar if and only if G contains no subgraph that is a subdivision of K_4 or $K_{2,3}$. (For the definition of outerplanar graphs see the TD on planar graphs).

2 Menger from Kőnig

Show that Kőnig's theorem implies Menger's theorem. In particular, show how a polynomial-time algorithm that decides if a bipartite graph has a matching of size at least k can be used to obtain a polynomial-time algorithm that decides for two vertices s, t of a graph G whether there exist at least k disjoint paths from s to t. (Reminder: in class we saw the opposite direction, namely, how Menger's theorem implies Kőnig's theorem.)

3 Rates of growth

Asymptotically, how many graphs on n vertices are there in the following classes? For classes marked with (*), give an upper bound (because a lower bound is harder to show).

1. All graphs
2. Forests(*)
3. Split graphs
4. Bipartite graphs
5. Chordal graphs
6. Interval graphs(*)
7. Planar graphs(*)

4 Brooks and bipartiteness

Let G be a connected graph with n vertices, m edges, and maximum degree 3 that is not a K_4. Show that G contains a bipartite subgraph with at least $m - \frac{n}{3}$ edges.

5 Cobipartite graphs are perfect

Prove that for all G, if $\overline{G}$ is bipartite, then G is perfect. Do not use the perfect graph theorem! (otherwise this is too easy)