TD 1: Introduction

1 Enumeration

List all non-isomorphic graphs on 2,3,4, and 5 vertices.

2 Connected Complements

Prove that for all graphs $G = (V, E)$, at least one of $G, \overline{G}$ is connected.

3 Connected Complements II

Show that if for a graph $G = (V, E)$ we have $\mathrm{diam}(G) \geq 3$, then $\mathrm{diam}(\overline{G}) \leq 3$.

4 Many edges connect the graph

Show that for any n-vertex graph $G = (V, E)$ with m edges, if $m > \binom{n-1}{2}$, then G is connected. Is this bound **sharp**? (meaning, is the claim false if we decrease the right-hand-side by 1?)

5 Walks and Adjacency Matrices

A **walk** is a path which is allowed to repeat vertices. Show that if A is the adjacency matrix of a graph G, then for all positive integers k we have that $A^k[i, j]$ is equal to the number of distinct walks of length exactly k from i to j in G.

6 Min degree to Path

Show that if all vertices of G have degree at least k, then G contains a path of length at least k.

7 Odd degrees

Prove that if a graph G contains exactly two vertices of odd degree, then they are connected by a path.

8 Ramsey

Prove that in any group of 6 people, there are either 3 people who all know each other or 3 people who do not know each other. Show that this is false for groups of 5 people.

Generalization: prove that for all k, in any group of 4^k people, there are either at least k who all know each other, or at least k who do not know each other.