## Subject Index

Bold numbers refer to pages where items are introduced.

0,1 polytope $\mathbf{7 5}$
$\{0,1\}$-valued vector $\mathbf{1 1}$
0,1 vector 11
0-join 1112-1113
1-bend cut 1324
1-cycling matroid 1421,1461
1 -factor $\equiv$ perfect matching 414415, 425-428, 431-436
1-factor theorem, Tutte's $\sim 414-415$, 425, 435-436
1-flowing matroid 1421,1461
1-join 1113
1-skeleton 65
1-tree 985-986
shortest 985
1-tree, directed ~ 993
shortest 993

2-commodity flow $1251-1265,1414$
characterization 1252-1254
2-commodity flow, half-integer $\sim$ 1251-1256
algorithm 1254
2-commodity flow theorem, Hu's ~ 1253-1254
2-connectivity 243
algorithm 243
2-cycling matroid 1421-1422
2-edge-connected component 247-248 algorithm 248
2-edge-connector 1062-1063
minimum-size 1062-1063
formula 1062-1063
2-edge cover 531-532, 534
minimum-size 531-532
algorithm 532
min-max 532
minimum-weight 534
$\min -\max 534$
2-edge cover, simple $\sim \mathbf{5 3 5}-536$
minimum-size $535-536$
algorithm 535
min-max 535
minimum-weight $535-536$
algorithm 536
2-edge cover polyhedron 533-534
2-edge cover polytope, simple $\sim \mathbf{5 3 6}$
2-factor 527-528, 531, 545, 986-987, 1456
algorithm 528
characterization 527-528
complexity 545
minimum-weight 528, 531, 986-987
algorithm 528
$\min -\max 531$
2-factor polytope $\mathbf{5 3 0}$
diameter 530
facets 530
2-flowing matroid 1421-1422
2-heap $98-99,128-129$
2-join 1112-1115
2-join, special ~ $\mathbf{1 1 1 4}$
2-matching 341, 520-521, 523-526, 531-532
maximum-size $520-521,524-526$, 531-532
algorithm 521
min-max $\quad 520-521$
maximum-weight 523-524
algorithm 523
$\min -\max \quad 523$
2-matching, perfect $\sim \mathbf{5 2 1}, 524$
characterization 521
complexity 521
minimum-weight 524
min-max 524
2-matching, perfect simple $\sim \equiv$ 2-factor
2-matching, simple $\sim$ 526-531, 535
maximum-size $526-528,535$
algorithm 528
min-max 526-527
maximum-weight 531 min-max 531
2-matching, simple perfect $\sim \equiv$ 2 -factor
2-matching, square-free $\sim 341$
2-matching, triangle-free ~ 539-544
maximum-size 542-544
2-matching, triangle-free perfect $\sim 544$ algorithm 544
2-matching lattice, perfect $\sim \mathbf{6 4 7}$
2-matching polytope $\mathbf{5 2 2}, 560$
facets 560
vertices 560
2-matching polytope, perfect ~522524
2-matching polytope, simple $\sim \mathbf{5 2 8}$ 531
facets 530
2-matching polytope, triangle-free $\sim$ 539-544
facets 544
2-matching space, perfect $\sim$ 646-647
2-packing 502
2-stable set 531-532, 578, 1091
maximum-size 531-532
algorithm 532
min-max 532
maximum-weight 578, 1091
algorithm 1091
min-max 578
2-stable set polyhedron $\mathbf{1 0 9 1}$
2-vertex-connectivity 243
algorithm 243
2-vertex-connector 1077-1078
minimum-size 1077-1078
$\min -\max \quad 1077-1078$
2-vertex cover 520-521, 531-532, 556557, 1094
minimum-size 520-521, 531-532
algorithm 521
min-max 520-521
minimum-weight 556-557, 1094
algorithm 1094
min-max 557
2-vertex cover polyhedron $\mathbf{1 0 9 4}$
3 -commodity flow 1230-1232, 1244, 1270-1275, 1295

3-cycling matroid 1422-1423
3 -dimensional matching problem 408
3-flow conjecture 472, 1454
3 -flow conjecture, weak $\sim 473,1454$
3-flowing matroid 1422-1423
3 -SAT $\equiv 3$-satisfiability problem
3-satisfiability problem 46
NP-completeness 46
4-cycling matroid 1423-1424
4-flow conjecture 472, 498, 645, 1426, 1454
4-flowing matroid 1423-1424
$4 \mathrm{CC} \equiv$ four-colour conjecture 1085
5-flow conjecture 472, 646, 1453
$\infty$-cycling matroid 1420, 1423-1424
$\infty$-flowing matroid 1420, 1423-1424
$\infty$-flowing matroid, integer $\sim 1420$
Ackermann function $\mathbf{8 6 4}$
Ackermann function, inverse $\sim \mathbf{8 6 4}$
active constraint $\mathbf{6 4}$
active inequality 63
active vertex 156
acyclic digraph 34, 89-90, 116-117, 154-155, 159-160, 218-224, 227, 229, 233, 913, 951, 964, 969, 1076, 1225, 1243-1245, 1307, 1325, 1337
acyclic subgraph polytope $\mathbf{9 5 2}$
adding ear 93, 252, 511
address 48
adjacency matrix
of digraph 35
of graph 28
adjacent faces $\mathbf{2 6}$
adjacent pairs of trees $\mathbf{8 9 2}$
adjacent spanning trees 207
adjacent vertices
of digraph 29
of graph 17
of polyhedron 65
affine halfspace $5 \mathbf{5 9}, \mathbf{6 0 7}$
affine halfspace, rational $\sim \mathbf{6 0 7}$
affinely independent vectors 13
algebraic matroid 656-657, 675-679, 753-754, 765
algorithm, efficient ~ 39
algorithm, good $\sim 39$
algorithm, linear-time $\sim 47$
algorithm, polynomial-time ~39-40
algorithm, semi-strongly polynomial-time $\sim 48$
algorithm, strongly polynomial-time $\sim$ 47-48, 69-70
algorithm, weakly polynomial-time $\sim$ 48
all-pairs minimum-size cut 248-251
all-pairs minimum-size cut problem 248
all-pairs shortest paths 91-94, 104105, 110-111, 113-114, 122, 125, 127, 129, 517
arbitrary-length 110-111, 113-114, 517
complexity 113
planar 113-114
undirected 517
algorithm 517
complexity 517
nonnegative-length 104-105
complexity 104-105
planar 105
complexity 105
unit-length 91-93 algorithm 91-92 complexity 93
zero-length 94 complexity 94
almost bipartite graph 336, 1206
almost regular edge set 268
$\alpha$-critical graph 1199
$\alpha$-diperfect digraph 1131, 1458
( $\alpha, \omega$ )-graph 1117
alphabet 40
alternating forest, $M-\sim 420$
alternating walk, $M$-~ 416
alternating walk, $S-\sim 1208$
amalgam 1395
of clutters 1395
of graphs 1130-1131
antiblocker 1430-1431
antiblocking body $\mathbf{6 7}$
antiblocking hypergraph 1430-1431
antiblocking pair 67
antiblocking pair of polyhedra $\mathbf{6 7}$
antiblocking polyhedron 67, 70-71, 82-83
antiblocking set $\mathbf{6 7}$
antiblocking type, polyhedron of $\sim \mathbf{6 7}$
antichain 217-236, 1026-1027
maximum-size 218 min-max 218
maximum-weight 220 min-max 220
antichain polytope 222
antichains, covering by $\sim$ 217, 220 min-max 217, 220
antichains, union of $\sim$ 226, 235, 1027 min-max 226
antihole 1085, 1107
antihole, odd $\sim$ 1085, 1107
apex graph 1310, 1350
arborescence 34, 893, 902
shortest 902 min-max 902
arborescence, mixed $r-\sim 926$
arborescence, partial $r$-~ 918
arborescence, $r$-~ 34, 254, 893-897, 902-903, 972, 1024, 1399
shortest 893-897, 902-903, 972, 1024
algorithm 893-895 complexity 902 min-max 896
arborescence in branching 1013
arborescence polytope 901-902
arborescence polytope, $r-\sim$ 897-899, 907
arborescence rooted at vertex $\mathbf{8 9 3}$
arborescence theorem, optimum $\sim$ 896, 898, 972, 1024, 1399
arborescences, capacitated disjoint $r$-~ 922
complexity 922
arborescences, covering by $r-\sim$ 911913
min-max 912-913
arborescences, disjoint $\sim 905,908$, 923-926
min-max 905,908
arborescences, disjoint $r$-~ 905-907, 918-922, 925, 974, 1078-1079
algorithm 918-921
complexity 921-922
min-max 905-907
arborescences, union of $\sim$ 916-918
arborescences, union of $r-\sim$ 913-915
min-max 913, 915
arborescences theorem, Edmonds'
disjoint ~ 905, 974, 1047, 1399
arboricity $\mathbf{8 7 9}$
arboricity, fractional $\sim 891$
arc 28
of digraph 28
arc, backward $\sim 31$
arc, forward $\sim 31$
arc-connected digraph, $k$-~ 238, 1051
minimum-size 1051
arc-connected orientation, $k-\sim$ 10441046
algorithm 1045
characterization 1044-1046
arc-connectivity 238, 243-244, 246247, 254-255, 1044-1046, 1048, 1051, 1058-1062
algorithm 244
complexity 246-247
arc-connector, $k$-~ 1058, 1060-1061
minimum-size 1060-1061
algorithm 1061
min-max 1060
arc-disjoint paths 132, 906, 1307
arc-disjoint paths problem 1223
arc-disjoint paths problem, $k \sim 1223$
arc-disjoint $s-t$ paths 132, 134-140, 142-147, 151
algorithm 134-138
complexity 138-139
min-max 132
planar 139-140 complexity 139-140
arc-disjoint subgraphs $\mathbf{3 0}$
arc-disjoint walks 32
arcs, disjoint $\sim 29$
arcs, parallel $\sim \mathbf{2 9}$
arithmetic operations, elementary $\sim$ 39
assignment, job ~ 428-429
assignment polytope 307-308
diameter 307-308
assignment problem 288, 290-300
algorithm 288
complexity 290
history 292-300
simplex method 290-291
assignment problem, bottleneck $\sim 291$
associated bipartite graph 1444
asymmetric postman problem 518
asymmetric traveling salesman polytope 992-996, 1003
adjacency 994
diameter 994
dimension 994
facets 992
asymmetric traveling salesman problem 981-982, 992-1004
NP-completeness 982
atom of lattice $\mathbf{6 6 8}$
augmenting algorithm, flow-~ 151
augmenting algorithm, matching-~

## 418

augmenting path 134, 151
augmenting path, $f-\sim 151$
augmenting path, fattest $\sim 159$
augmenting path, flow-~ 151
augmenting path, $M-\sim$ 259-260, 263264, 413
augmenting path, matching-~ 259
augmenting path, $S$-~ 1208
automorphism 1169
$b$-bicritical graph $\mathbf{5 6 0}$
$b$-critical graph $\mathbf{5 5 9}$
b-detachment $\mathbf{7 0 4}$
$b$-edge cover $\mathbf{3 4 7}-354,361,575-583$
bipartite 347-354, 361
minimum-size $348,352,361$
algorithm 352, 361
min-max 348
minimum-weight 348, 352-353
algorithm 352-353
min-max 348
minimum-size 351-352, 575-576, 578
algorithm 576
min-max 576, 578
minimum-weight 577-578
algorithm 577-578
min-max 577-578
b-edge cover, capacitated $\sim 350-353$, 579-580, 583
bipartite 350-353
minimum-size 350-351
min-max 350-351
minimum-weight 351-353 algorithm 351-353 min-max 351
minimum-size 579-580, 583
algorithm 580
min-max 579-580
minimum-weight 580
algorithm 580
min-max 580
$b$-edge cover, simple ~ 349-354, 581582
bipartite 349-354
minimum-size 349-350
min-max 349
minimum-weight 350-353
algorithm 350-353
min-max 350
minimum-size 581-582
algorithm 581-582
min-max 581-582
minimum-weight 581
min-max 581
$b$-edge cover polyhedron 348, 576-577
bipartite 348
$b$-edge cover polytope, $c$-capacitated $\sim$ 580
$b$-edge cover polytope, simple $\sim \mathbf{3 5 0}$, 581
bipartite 350
b-factor 340-343, 358, 569-574, 621
algorithm 572
bipartite 340-343, 358
algorithm 342-343
characterization 340
complexity 358
minimum-weight 341-343
algorithm 342-343
min-max 341
characterization 570
minimum-weight 571-572
algorithm 572
min-max 571
$b$-factor polytope 570-571
b-matching 337-347, 351-356, 358362, 546-576
bipartite 337-347, 353-356, 358362
maximum-size 338, 342-343, 358
algorithm 342-343
complexity 358
min-max 338
maximum-weight 337-338, 342343, 355-356
algorithm 342-343
complexity 355-356
min-max 338
maximum-size 351-352, 546-547,
556-557, 575-576
min-max 546-547, 557
maximum-weight 550-559, 561
algorithm 554-556, 561
complexity 559
min-max 550-553, 558
$b$-matching, capacitated $\sim$ 341-343, 357-358, 361, 562-568, 583
bipartite 341-343, 357-358
maximum-size 341-343, 358
algorithm 342-343
complexity 358
min-max 341-342
maximum-weight 342-343, 357
algorithm 342-343
complexity 357
min-max 342
maximum-size $\quad 562-564,567,583$
min-max 562-564
maximum-weight 566-567
algorithm 567
min-max 566
$b$-matching, capacitated perfect $\sim$ 342-343, 358, 564, 567
bipartite 342-343, 358
characterization 342
complexity 358
characterization 564
minimum-weight 567 algorithm 567
$b$-matching, perfect $\sim 338,343,358$, 547, 553-554, 556-557, 567-568
bipartite 338, 343, 358
characterization 338
complexity 358
minimum-weight 343
algorithm 343
characterization 547, 557
minimum-weight 553-554, 556 algorithm 556 min-max 553-554
$b$-matching, perfect simple $\sim \equiv$ $b$-factor
b-matching, simple $\sim 339-343,354$, 358, 569-574, 582
bipartite $339-343,354,358$ maximum-size $339,342-343$, 358
algorithm 342-343
complexity 358
min-max 339 maximum-weight $340-343$
algorithm 342-343
min-max $\quad 340-341$
maximum-size $569,572-573,582$ algorithm 572 min-max 569
maximum-weight 571-573 algorithm 571-572 min-max 571
$b$-matching, simple perfect $\sim \equiv$ $b$-factor
$b$-matching polytope 338-339, 547553, 557, 559-561
adjacency 549
bipartite 338-339
diameter 549
facets 559
$b$-matching polytope, $c$-capacitated $\sim$ 342, 564-567
bipartite 342
facets 567
$b$-matching polytope, $c$-capacitated perfect $\sim \mathbf{5 6 5}$
$b$-matching polytope, fractional $\sim 561$
vertices 561
$b$-matching polytope, perfect $\sim \mathbf{5 4 9}$, 553-554
$b$-matching polytope, simple $\sim \mathbf{3 4 0}$, 570-571, 574
adjacency 574
bipartite 340
facets 574
b-transportation 343-346, 356-357, 361-377
minimum-cost $344-346,356-357$, 361-377
algorithm 344-346 complexity 356-357
$b$-transportation, capacitated $\sim 357-$ 358, 361-377
minimum-cost $357-358,361-377$ complexity 357-358
b-transshipment 173-175, 182-184, 186-189, 191-192, 345-346
characterization 174-175
minimum-cost 182-183, 186-189, 191-192, 345-346 algorithm 182-183, 186-189 complexity 191 min-max 191-192
$b$-transshipment polytope 207-210
$b$-transshipment space 208
dimension 208
backward arc 31
bad $K_{4}$-subdivision 1195
balanced bipartite graph, totally $\sim$ 1444
balanced hypergraph 1439-1451
characterization 1440-1443
balanced hypergraph, totally $\sim 1446{ }^{-}$ 1447
balanced matrix 1439,1447
balanced matrix, totally $\sim 1444-1447$
barrier 427-428
barrier, simple ~ $\mathbf{6 2 4}$
base 669-671, 689-690, 692, 699, 722, 728-729
exchange properties $669-671,722$, 728-729
minimum-weight 689-690, 692, 699
algorithm 689-690
min-max 692
of callection of pairs in matroid

## 746

of element of polymatroid $\mathbf{7 7 9}$
of matroid 651, 662, 669-671, 728729
of subset of matroid 651
base, common $\sim 701,710,715,740-$ 743
characterization 701
minimum-weight 710,715
algorithm 710
$\min -\max 715$
base covering theorem, matroid $\sim$ 727, 729
base orderable matroid, strongly $\sim$ 738-743
base packing theorem, matroid $\sim \mathbf{7 2 7}$
base polyhedron $\mathbf{8 4 1}$
base polytope $\mathbf{6 9 2}-693,730-731,734$, 767, 787, 841-842
base polytope, common $\sim \mathbf{7 1 5}, 719-$

$$
720,741-743
$$

dimension 719
base vector $\mathbf{7 6 7}, \mathbf{7 7 4}$
base vector, unit ~ $\mathbf{1 2}$
bases, covering by $\sim$ 726-727, 729, 732, 735-736
algorithm 732, 735-736
min-max 727, 729
bases, covering by common $\sim 741-743$

$$
\min -\max \quad 741
$$

bases, disjoint $\sim$ 727, 732, 734, 736
algorithm $732,734,736$
min-max 727
bases, disjoint common $\sim$ 740-741
min-max 740
basic path-matching $\mathbf{7 6 3}$
Bellman-Ford method 109-110, 122125
bend cut, 1-~ 1324
Berge graph 1107, 1112, 1124, 1127
bibranching, $R-S \sim$ 934-945, 972, 1024
minimum-size $934-935$
algorithm 935
min-max 935
shortest 935-937, 972, 1024
algorithm 937
min-max 936-937
bibranching polytope, $R-S \sim \mathbf{9 3 7}$, 942
bibranching theorem, optimum $\sim 937$, 972, 1024
bibranchings, disjoint $R-S \sim 940-$ 944, 974
min-max 941-942
bibranchings, disjoint $R-S-\sim 942$
algorithm 942
bibranchings theorem, disjoint $\sim$ 941943, 974
bicircular matroid $\mathbf{7 4 3}$
bicolourable hypergraph 1443
bicolouring number 1118
biconnector, $R-S \sim$ 928-930, 944
minimum-size 929
min-max 929
shortest 928-930
algorithm 930
min-max $929-930$
biconnector polytope, $R-S \sim \mathbf{9 2 9}$ 930
biconnectors, disjoint $R-S \sim 931-$ 934, 944
algorithm 933
min-max 933
bicritical graph 503, 614, 619
bicritical graph, $b-\sim \mathbf{5 6 0}$
bicut, $R-S \sim 935,940-943,972$, 974, 1024
minimum-size $940-943,974$ min-max 941-942
bicuts, disjoint $R-S \sim 937,972,1024$
$\min -\max 937$
bidirected graph 594-608, 1201-1203
bidirected graph, claw-free $\sim 1217$
biforest, $R-S \sim$ 930-931, 944-945
longest 930-931
algorithm 931
min-max 930
biforest polytope, $R-S \sim \mathbf{9 3 1}$
biforests, covering by $R-S \sim 934$, 944-945
algorithm 934
min-max 934
bifurcation, $R-S \sim$ 937-940, 944945, 1016
longest 938-940 algorithm 940 min-max $938-940$
maximum-size 937-938 min-max 938
bifurcation polytope, $R-S \sim \mathbf{9 4 0}$, 944
bifurcations, covering by $R-S \sim 943-$ 945
algorithm 944
min-max $943-944$
bijection 13
bimatroid 671
binary hypergraph 1406-1418

1814 Subject Index
binary ideal hypergraph 1408-1409, 1460-1461
binary matroid 655-656, 1406-1407, $1415,1420-1427,1456,1461$
binary matroid, cycle in $\sim \mathbf{6 5 5}$
binary Mengerian hypergraph 14091415
characterization 1409-1412
bipartite edge-colouring 1136
bipartite edge set $\mathbf{1 3 2 6}$
bipartite graph 24, 259-377, 959-960, 1135-1137
bipartite graph, almost $\sim 336,1206$
bipartite graph, complete $\sim \mathbf{2 4}$
bipartite graph, near-~ 1217
bipartite graph, strongly $\sim 1328$, 1333-1334, 1414
bipartite graph, weakly $\sim 1326-1327-$ 1329, 1334-1341, 1392
bipartite signed graph, evenly $\sim 1331$, 1340
characterization 1340
bipartite signed graph, strongly $\sim$ 1330-1333
characterization 1333
bipartite signed graph, weakly ~ 1330-1331, 1340
characterization 1340
bipartite subgraph polytope 1326, 1350
facets 1350
Birkhoff's theorem 302-303
bisubmodular function 851
bisupermodular function 851
bit 38
block 242-243, 633
algorithm 242-243
block, isolated ~ 1077
block, pendant ~ 1077
blocker 1377
blocking collection of paths 135
blocking flow 154-156
blocking hypergraph 1377
blocking pair 66
blocking pair of polyhedra $\mathbf{6 6}$
blocking polyhedron $\mathbf{6 6}, 70,82$
blocking type, polyhedron of $\sim \mathbf{6 6}$
blossom, $M-\sim 416$
blow-up 1129
body, antiblocking ~ $\mathbf{6 7}$
body, convex ~ $\mathbf{5 9}$
bone 1211
Boolean expression 44
border, reduced $T$-~ $\mathbf{5 0 7}$
border, $T$-~ 501
Borůvka's method 859, 871-874
bottleneck assignment problem 291
bottleneck extremum 1380
bottleneck maximum 1379-1380
bottleneck minimum 1379-1380
bottleneck shortest path 117-118, 130
boundary square of polyomino $\mathbf{1 1 4 9}$
bounded face of planar graph 26
box 75
box-integer polyhedron 75, 1418
box-TDI $\equiv$ box-totally dual integral
box-totally dual integral 83
brace 614
branch-and-bound method 982-984, 996
branch-and-cut method 984
branching 34, 893, 895-896, 900-901, 909-911, 960
exchange properties 909-910
longest 895-896, 900-901 algorithm 895-896 min-max 900-901
branching, co~ 937, 942
branching, mixed ~ 926
branching polytope 901, 909
adjacency 901
facets 901
branchings, covering by ~ 908-909, 911, 922
complexity 922
min-max $908-909$
branchings, disjoint ~ 904-905, 922
characterization 904-905
complexity 922
branchings, union of $\sim$ 915-918
min-max $916-918$
branchings theorem, Edmonds' disjoint

$$
\sim 904
$$

breadth-first search 88
brick 614-617, 630-643, 647
brick decomposition 612-613
bridge
of graph 21
of matroid 653
bridgeless graph 21
Brooks' theorem 1086
bucket 102
bull 1121
bull-free graph 1121
$c$-capacitated $b$-edge cover polytope 580
$c$-capacitated $b$-matching polytope 342, 564-567
bipartite 342
facets 567
$\mathcal{C}$-cover 976
$c$-covering 36
$c$-covering, fractional $\sim 37$
$\mathcal{C}$-cut 976
c-packing $\mathbf{3 6}$
$c$-packing, fractional $\sim \mathbf{3 6}$
cactus 253
cap 1145
cap-free 1145
capacitated $b$-edge cover $\mathbf{3 5 0}-353$, 579-580, 583
bipartite 350-353
minimum-size 350-351
min-max 350-351
minimum-weight 351-353
algorithm 351-353 min-max 351
minimum-size 579-580,583
algorithm 580
min-max 579-580
minimum-weight 580
algorithm 580
$\min -\max 580$
capacitated $b$-edge cover polytope, $c$-~ 580
capacitated $b$-matching 341-343, 357358, 361, 562-568, 583
bipartite 341-343, 357-358
maximum-size $341-343,358$
algorithm 342-343
complexity 358
min-max 341-342
maximum-weight 342-343, 357
algorithm 342-343
complexity 357
min-max 342
maximum-size $562-564,567,583$ min-max $\quad 562-564$
maximum-weight 566-567 algorithm 567 min-max 566
capacitated $b$-matching polytope, $c$-~ 342, 564-567
bipartite 342
facets 567
capacitated $b$-transportation 357-358, 361-377
minimum-cost $357-358,361-377$ complexity 357-358
capacitated common transversal 407
capacitated disjoint $r$-arborescences 922
complexity 922
capacitated perfect $b$-matching 342343, 358, 564, 567
bipartite 342-343, 358 characterization 342 complexity 358
characterization 564
minimum-weight 567 algorithm 567
capacitated perfect $b$-matching polytope, $c$-~ 565
capacitated transportation 357-358, 361-377
minimum-cost $357-358,361-377$ complexity 357-358
capacity 13
of cut 149
of path $\mathbf{1 1 7}$
capacity, Shannon ~ 1167-1171, 1176-1178, 1184-1185
capacity function 13
capacity-scaling 159-160
Carathéodory's theorem 59, 63
cellularly embedded graph 1357
certificate 41
chain 217-236, 1026-1027
maximum-size 217 $\min -\max 217$
chain, maximal $\sim 235$
chain, symmetric $\sim 236$
chain of sets 10
chain polytope 221-222
chains, covering by $\sim 218$
$\min -\max 218$
chains, disjoint maximal $\sim 235$
min-max 235
chains, union of $\sim 228-229,1026-1027$
min-max $\quad 228-229$
chair 1121
chair-free graph $\mathbf{1 1 2 1}$
channel routing problem $\mathbf{1 3 2 3}$
characteristic cone 60
characterization, good $\sim 42-43$
checked graph 1121
$\chi$-diperfect digraph 1132
child 99
Chinese postman problem 487-488, 518-519
algorithm 487-488
complexity 488,518
history 519
windy postman problem 518
Chinese postman problem, directed $\sim$ 192, 518
Chinese postman problem, mixed $\sim$ 518
Chinese postman tour 487
chord
of circuit 20,1138
of path 19
chordal bipartite graph 1444
chordal graph 1138-1143
chordal graph, strongly $\sim 1142$
chordal graph, weakly $\sim 1148$
chordless circuit 20
chordless path 19
Christofides' heuristic for the symmetric traveling salesman problem $\mathbf{9 8 9}$
chromatic graph, $k-\sim 23,1083$
chromatic graph, $k$-edge- $\sim 24$
Chvátal comb inequality 988
Chvátal rank 607-608, 1098-1099
Chvátal rank, strong ~ 608
circle graph 1100,1121
circuit 20, 500-501, 746-747
in digraph 32
minimum-mean length 500-501 algorithm 500-501
of binary hypergraph 1409
of matroid $\mathbf{6 5 1}, 662-664,672$ shortest 672
circuit, chordless $\sim \mathbf{2 0}$
circuit, directed $\sim 32$
circuit, directed Hamiltonian $\sim 115$, 981
NP-completeness 115
circuit, even $\sim \mathbf{1 3 2 9}$
in bidirected graph 1201
circuit, Hamiltonian $\sim \mathbf{2 4}, \mathbf{3 4}, \mathbf{9 8 1}-$ 982, 996
longest 996
shortest 981-982
circuit, $k$-~ $\mathbf{2 0}$
circuit, odd $\sim$ 1326-1329-1341, 1414
in bidirected graph 1201
in signed graph 1414
circuit, shortest directed $\sim 94$
circuit, undirected $\sim 32$
circuit cone
of graph $\quad \mathbf{4 9 3}-498, \mathbf{6 0 5}, 1456$
of matroid 1424-1426
circuit cover, odd ~1327, 1329, 1335-1340, 1414
$\min -\max$ 1335-1340
circuit double cover 1427
circuit double cover conjecture 497, 645-646, 1427, 1456
circuit-free vertex set $\mathbf{8 7 0}-871$
circuit lattice of matroid 1425-1426
circuit space of matroid 1425
circuits, disjoint directed $\sim$ 958-959, 1368
complexity 959
planar 958
min-max 958
circuits, disjoint odd $\sim 1335-1340$
min-max 1335-1340
circuits, sums of $\sim 493-498,1424-1426$
in matroid 1424-1426
circular-arc graph 1100,1121
circular flow conjecture 473,1454
circulation 171-172, 175-191, 195197, 207
algorithm 175-176
characterization 171-172
minimum-cost $\quad 177-191,195-197$ algorithm 179-182, 189-190 complexity 190-191
simplex method 195
circulation, feasible $\sim \mathbf{1 7 8}$
circulation freely homotopic to $\mathbf{1 3 5 7}$, 1360
circulation problem, minimum-cost $\sim$ 177
circulation theorem, Hoffman's ~ 171-172, 1020
circulation theorem, homotopic $\sim$ 1357-1360
city $\mathbf{9 8 2}$
class 9
of partition 10
of splittable vertex 1210
claw 24, 1120
claw-free bidirected graph 1217
claw-free graph 1120, 1208-1217
claw-free graph, perfect ~ 1120
clique 23, 1083-1085, 1097, 11021185, 1458
in digraph 1131
in perfect graph 1106-1134, 1154, 1157, 1159
maximum-size 1106-1134, 1154 algorithm 1154
maximum-weight 1157, 1159 algorithm 1157, 1159
maximum-size 1084-1085, 11021185
NP-completeness 1084-1085
maximum-weight 1097, 1157
clique cover 1083
clique cover, minimum ~ 1083
clique cover number 1083
clique cover number, fractional $\sim$ 1096
clique cover number, fractional weighted

## $\sim 1097$

clique cover number, weighted $\sim \mathbf{1 0 9 7}$
clique inequality 1095-1096
clique number 23,1083
clique polytope 1088, 1110-1111
of perfect graph 1110-1111
clique tree inequality 987-989
closed curve 1352
closed curve, doubly odd $\sim 1367$
closed curve, simple $\sim$ 1321, 1352
closed directed walk 32
closed walk 20
clutter 1376
co-NP 42, 71-72
coarborescence, $r$-~ 941
cobranching 937, 942
cocircuit of matroid 653, 663-664
coclique $\equiv$ stable set
cocycle matroid 657-658
cographic matroid 657-658
collection 9
coloop of matroid $\mathbf{6 5 3}$
colour 23, 321, 465, 1083
colour, edge-~ 465
colour, have $\sim 321$
colour classes 24
colourable graph, 3-~ 1085-1087
colourable graph, $k$-~ 23, 1083
colourable graph, $k$-edge-~ 24, 465
colourable graph, $k$-list-edge-~ 335
colourable graph, $k$-vertex-~ 23, 1083
coloured tree 703
colouring 23, 1083-1088, 1098, 11011185, 1206-1207, 1458
colouring, edge-~ 23, 321-331, 333336, 465-484, 1016, 1136, 1455
bipartite 321-331, 333-336, 1016, 1136
algorithm 322-323, 333-334
complexity 334-335
min-max 321-322
complexity 466-467
history 482-484
NP-completeness 468-470
colouring, fractional edge- $\sim$ 474-478, 1455
complexity 477-478
min-max 474-475
colouring, $k$-~ 1083
colouring, $k$-edge-~ 321, 465
colouring, $k$-interval $\sim 1151$
colouring, list-~ 737-738, 892
of matroid 737-738
colouring, minimum ~ 23, 1083-1088, 1098, 1102-1185, 1206-1207
NP-completeness 1084-1085
of perfect graph 1106-1134, 11541155
algorithm 1154-1155
colouring, minimum edge-~ 24
colouring, minimum fractional $\sim 1096$, 1098
colouring, minimum fractional vertex-~ 1096, 1098
colouring, minimum fractional weighted ~ 1097
NP-completeness 1097
colouring, minimum vertex-~ 23, 1083-1088, 1098, 1102-1185, 1206-1207
NP-completeness 1084-1085
of perfect graph 1106-1134, 11541155
algorithm 1154-1155
colouring, minimum weighted $\sim 1096$ 1097, 1157-1159
NP-completeness 1096-1097
of perfect graph 1157-1159 algorithm 1157-1159
colouring, minimum weighted vertex-~ 1096-1097, 1157-1159
NP-completeness 1096-1097
of perfect graph 1157-1159 algorithm 1157-1159
colouring, supermodular ~ 849-851, 943
colouring, total ~ 482, 1455-1456
colouring, vertex-~ 23, 1083-1088, 1098, 1101-1185, 1206-1207
colouring number $\equiv$ vertex-colouring number 23, 1083
colouring number, edge- $\sim$ 23, 321, 465
colouring number, fractional $\sim 1096$
colouring number, fractional edge-~ 474
colouring number, fractional weighted $\sim$ 1097
colouring number, list-edge-~ 335, 482
colouring number, total $\sim \mathbf{4 8 2}$
colouring number, vertex-~ 23, 1083
colouring number, weighted $\sim 1096$
colouring theorem, Kőnig's edge-~ 321-322, 324-325, 331, 934, 1016, 1136, 1441
column generation technique $1245-$ 1247
column strategy 296
comb inequality 988
comb inequality, Chvátal ~ 988
combinatorics, polyhedral ~ 2, 6-7
history 6-7
commodity 1221
commodity flow, 2-~ 1251-1265, 1414
characterization 1252-1254
commodity flow, $3-\sim 1230-1232$, 1244, 1270-1275, 1295
commodity flow, half-integer 2-~ 1251-1256
algorithm 1254
commodity flow, $k$-~ 1221-1222
commodity flow problem, integer $k$-~ 1222
commodity flow problem, $k$-~ 1221
commodity flow problem, maximum-value $k$-~ $\mathbf{1 2 2 2}$
commodity flow problem, undirected $k$-~ 1222
commodity flow problem, undirected maximum-value $k$-~ $\mathbf{1 2 2 2}$
commodity flow theorem, Hu's 2-~ 1253-1254
common base $701,710,715,740-743$
characterization 701
minimum-weight 710, 715
algorithm 710 min-max 715
common base polytope 715, 719-720, 741-743
dimension 719
common bases, covering by $\sim$ 741-743 min-max 741
common bases, disjoint $\sim$ 740-741 $\min -\max 740$
common independent set 700-701, 705-724, 768, 1026
exchange property $721-722$
maximum-size 700-701, 705-707, 710, 1026
algorithm 705-707
complexity 707, 710
min-max 700-701
maximum-weight 707-712, 714-715
algorithm $\quad 707-712$
min-max 714-715
of three matroids 700, 707
NP-completeness 700, 707
common independent set, extreme $\sim$ 707
common independent set augmenting algorithm 705-706
common independent set augmenting algorithm, maximum-weight $\sim$ 707-709
common independent set polytope 712-714-719, 741-743
facets 718-719
common independent sets, covering by ~ 739-740
min-max 740
common partial transversal 393-395, 397-399
maximum-size 394 min-max 394
maximum-weight 397-399
algorithm 397 min-max 398-399
common partial transversal polytope 399-400
common partial transversals, covering by $\sim 402-403,406$
min-max 402
common spanning set $701,716,741$
minimum-size 701 min-max 701
minimum-weight 716 min-max 716
common spanning set polytope 715716
common spanning sets, disjoint $\sim 741$
min-max 741
common system of restricted representatives 407
characterization 407
common transversal 393-409, 703
algorithm 394
characterization 393-394
exchange property 407-408
minimum-weight 395-397 algorithm 396
min-max 396-397
NP-completeness 408
common transversal, capacitated $\sim$ 407
common transversal polytope 401-402
common transversals, covering by $\sim$ 405-406
min-max 405-406
common transversals, disjoint $\sim 402$ 405
min-max 402-403
comparability graph 1137-1138, 1151
comparability graph, p-~ 1149
comparable sets 10, 1446
complement of graph 18
complementary graph 18
complementary slackness 63
complete bipartite graph 24
complete directed graph 30
complete graph 18
complete problem, NP-~ 43-44, 72
component
of graph $\mathbf{2 0}, 90-91,94-95$
algorithm 90-91
complexity 94-95
of hypergraph $\mathbf{3 6}$
of vector $\mathbf{1 1}$
component, 2-edge-connected $\sim 247$ 248
algorithm 248
component, connected $\sim$
of graph $\mathbf{2 0}, 90-91,94-95$
algorithm 90-91
complexity 94-95
of hypergraph $\mathbf{3 6}$
component, even $\sim \mathbf{2 0}$
component, inverting ~ 469
component, $k$-connected $\sim \mathbf{2 4 2}$
component, $k$-edge-connected $\sim 248$
component, marginal ~ $\mathbf{1 0 7 0}$
component, odd $\sim 20,413$
component, splitting ~ 469
component, weak $\sim 208$
component of digraph, strong $\sim \mathbf{3 2}$
component of digraph, strongly connected $\sim 32$
component of digraph, weak $\sim 32$
component of digraph, weakly connected ~ 32
component of hypergraph, nontrivial $\sim$ 757
concatenation of walks 19,31
concave-cost flow 196-197
cone 60
cone, convex $\sim \mathbf{6 0}$
cone, finitely generated $\sim \mathbf{6 0}$
cone, polar $\sim 65$
cone, polyhedral $\sim \mathbf{6 0}$
cone generated by $\mathbf{6 0}$
conformal hypergraph 1430-1431
conjugate partition 230
connect vertices, edge $\sim \mathbf{1 7}$
connected component
of graph 20, 90-91, 94-95
algorithm $90-91$
complexity 94-95
of hypergraph $\mathbf{3 6}$
connected component, 2-edge-~ 247 248
algorithm 248
connected component, $k$-edge-~ $\mathbf{2 4 8}$
connected digraph, $k-\sim$ 238, 1050 1051
minimum-size 1050-1051
connected digraph, $k$-arc-~ 238, 1051
minimum-size 1051
connected digraph, $k$-vertex-~ 238, 1050-1051
minimum-size 1050-1051
connected digraph, source-sink $\sim$ 964-967, 972-976
connected digraph, strongly $\sim \mathbf{3 2}, 93$
connected digraph, strongly $k-\sim \mathbf{2 3 8}$, 1051
minimum-size 1051
connected digraph, weakly ~ $\mathbf{3 2}$
connected graph 20
connected graph, $k-\sim \mathbf{2 3 7}, 1049-1050$
minimum-size 1049-1050
connected graph, $k$-edge-~ 238, 1050
minimum-size 1050
connected graph, $k$-vertex-~ $\mathbf{2 3 7}$, 1049-1050
minimum-size 1049-1050
connected graph, $r$-edge-~ $\mathbf{1 0 5 5}$, 1067
connected hypergraph $\mathbf{3 6}$
connected matroid 653, 698
connected orientation, $k$-arc-~ 1044 1046
algorithm 1045
characterization 1044-1046
connected orientation, strongly $\sim$ 1037-1040, 1048
algorithm 1037-1038
characterization 1037-1040
connected orientation, strongly $k-\sim$ 1044-1046
algorithm 1045
characterization 1044-1046
connected subgraph, $k-\sim 991$
shortest 991
connected vertices 17, 29
connecting edge sets, path $\sim \mathbf{1 2 6 3}$
connectivity 237-238-243, 253-255, 1049-1051, 1074-1078, 1458
algorithm 239-241
complexity 241
connectivity, 2-~ 243
algorithm 243
connectivity, 2-vertex-~ 243
algorithm 243
connectivity, arc-~238, 243-244, 246247, 254-255, 1044-1046, 1048, 1051, 1058-1062
algorithm 244
complexity 246-247
connectivity, edge-~ 237-238, 244251, 253-255, 1037-1040, 10441046, 1048, 1050, 1055-1057, 1062-1074, 1458
algorithm 244-246
complexity 246-247
connectivity, vertex-~ 237-238-243, 253-255, 1049-1051, 1074-1078, 1458
algorithm 239-241
complexity 241
connectivity augmentation 969, 10581079, 1457
for hypergraphs $\mathbf{1 3 8 2}$
NP-completeness 969, 1062, 10661067, 1079
connectivity augmentation, strong $\sim$ 969-973
algorithm 971-972
connectivity augmentation problem, strong ~ 969
connector 855
connector, 2-edge-~ 1062-1063
minimum-size $1062-1063$ formula 1062-1063
connector, 2-vertex-~ 1077-1078
minimum-size 1077-1078 min-max 1077-1078
connector, $k$-arc-~ 1058, 1060-1061
minimum-size 1060-1061 algorithm 1061 min-max 1060
connector, $k$-edge-~ 1062, 1065-1066 minimum-size 1065-1066 algorithm 1065 min-max 1065-1066
connector, $k$-vertex-~ 1074-1075, 1077
minimum-size 1074-1075 min-max 1074-1075
connector, $r$-edge-~ $\mathbf{1 0 6 7}$
connector, $s-t \sim 203$
connector, strong ~ 969-980, 1024
minimum-size 972 min-max 972
shortest 969-973, 1024 algorithm 971-972 min-max 971-972
connector polytope $\mathbf{8 6 3}, 878,881-882$, 884-887
facets 863
connector polytope, $s-t \sim$ 203-204 dimension 203
connectors, disjoint $\sim$ 877-880, 888889
algorithm 879-880, 888-889
min-max 877-878
connectors, disjoint strong ~ 973-976 algorithm 975-976 min-max 973-974
connects vertices, arc $\sim 29$
connects vertices, path $\sim 19,31$
conservation law, flow $\sim 148$
conservative function 494
contains 18
contractible to $K_{4}$, oddly $\sim 503$
contracting arc 35
contracting edge
in pair $G, T \quad 504$
of graph 25
of signed graph 1202, 1330
contracting elements of matroid 653
contracting vertex of hypergraph 1376
contracting vertex set in digraph 35
contracting vertex set in graph $\mathbf{2 5}$, 416
contraction, $\mathcal{F}-\sim \mathbf{6 1 0}$
contraction of hypergraph 1376
contrapolymatroid, extended $\sim \mathbf{7 7 4}$
contrapolymatroid intersection 797799, 818-819, 837
convex body 59
convex cone 60
convex-cost flow 196
convex hull 59
convex polyomino, horizontally $\sim$ 1149
convex polyomino, orthogonally $\sim$ 1149
convex subset of partially ordered set $\mathbf{1 0 2 8}$ of $\mathbb{R}^{n} \quad 59$
coparallel elements of matroid 653
copartition 838, 841, 1047
copartition, proper $\sim \mathbf{8 3 8}$
corner square of polyomino $\mathbf{1 1 4 9}$
coroot 942
correct word 45
corresponding walk 1214
cost 13
of circuit 1188
of circulation 177
of edge 1188
of family of vertices, edges, and odd circuits 1188
of flow 177
of transshipment $\mathbf{1 7 7}$
of vertex 1188
cost $b$-transshipment, minimum-~ 182-183, 186-189, 191-192, 345346
algorithm 182-183, 186-189
complexity 191
min-max 191-192
cost function 13, 63
cost transshipment, minimum-~ 182-
183, 186-189, 191-192, 345-346
algorithm 182-183, 186-189
complexity 191
min-max 191-192
cover 9, 17, 29, 668
cover, $\mathcal{C}$-~ 976
cover, $F$-~ 1203
cover, matroid $\sim$ 756-757
cover, $w$-~ 1188
cover in partially ordered set $\mathbf{2 3 4}$
covering 36
covering, $c$-~ 36
covering, fractional $\sim 36$
covering, fractional $c$-~ $\mathbf{3 7}$
covering, $k$-~ $\mathbf{3 6}$
covering by antichains 217, 220
min-max 217, 220
covering by bases $726-727,729,732$, 735-736
algorithm 732, 735-736
min-max 727, 729
covering by branchings $908-909,911$, 922
complexity 922
min-max 908-909
covering by chains 218
min-max 218
covering by common bases 741-743
min-max 741
covering by common independent sets 739-740
min-max 740
covering by common partial transversals 402-403, 406
min-max 402
covering by common transversals 405406
min-max 405-406
covering by directed cuts 218
acyclic 218
$\min -\max 218$
covering by forests $878-879,888-890$
algorithm 888
complexity 889-890
min-max 879
covering by independent sets 726-727, 729, 732, 735-736
algorithm 732, 735-736
min-max 727,729
covering by matching forests 1016
$\min -\max 1016$
covering by partial transversals 386387
min-max 386
covering by paths 219, 222-224
algorithm 222-224
min-max 219
covering by perfect matchings 329-331
bipartite 329-331
min-max 329-330
covering by $r$-arborescences 911-913 min-max 912-913
covering by $R-S$ biforests 934, 944945
algorithm 934
min-max 934
covering by $R-S$ bifurcations 943-945
algorithm 944
min-max 943-944
covering by $s-t$ paths 219-221
acyclic 219-220
min-max 219-220
min-max 220-221
covering problem, set $\sim 1438$
covers vertex, edge $\sim \mathbf{1 7}$
covers vertex, matching ~ 413
critical edge 1133
critical graph, $\alpha-\sim 1199$
critical graph, $b-\sim 559$
critical graph, factor-~ 424-425-426, 446, 544-545
critical graph, $P$-~ 544
critical hypergraph 1409
critical vertex set, $\mathcal{F}$-~ $\mathbf{5 4 5}$
critically imperfect graph $\equiv$ minimally imperfect graph 1107
cross 1302, 1306
cross-free collection of cuts 488, 610
cross-free cuts $\mathbf{6 1 0}$
cross-free family $\mathbf{3 7}, \mathbf{2 1 4}-216, \mathbf{8 4 2}$, 1021-1022
crosses vertex pair, edge pair ~ 1305
crossing family 838-851, 976-980, 1018-1023
crossing submodular function 838, 1018
crossing subsets 1291
crossing supermodular function $\mathbf{1 0 2 2}$
crossing system of curves, minimally $\sim$ 1353
cubic graph 17, 415, 432, 434
Cunningham-Marsh formula 440-441443
curve 1361
curve, closed ~ 1352
curve, doubly odd closed $\sim 1367$
cut 21, 33, 244-246, 253-254, 486, 1328, 1342, 1345-1350
maximum-capacity $486,1345-1350$ approximative algorithm 13451348
planar 486 algorithm 486
maximum-size 1328,1350 complexity 1350 NP-completeness 1328
minimum-capacity 253-254
minimum-size 244-246 algorithm 244-246
cut, 1-bend ~ $\mathbf{1 3 2 4}$
cut, all-pairs minimum-size $\sim 248-251$
cut, $\mathcal{C}$-~ 976
cut, $D_{0-\sim}$ 970-976
minimum-capacity 974
min-max 974
minimum-size 973-976 $\min -\max \quad 973-974$
cut, directed ~33, 116, 218-220, 946-968, 972, 1020, 1024, 1399
acyclic 219-220
maximum-size 219-220
min-max $\quad 219-220$
minimum-capacity 966 -967
minimum-mean capacity 968
minimum-size $962-968$ min-max $967-968$
source-sink connected 966-967 minimum-capacity $966-967$ min-max $\quad 966-967$ minimum-size 966 min-max 966
cut, fundamental $\sim 449,499$
cut, $k$-~ 21, 33
cut, $k$-vertex-~ 22, 33
cut, maximum $\sim \equiv$ maximum-size cut
cut, minimum $\sim \equiv$ minimum-size cut 238
cut, minimum vertex- $\sim \equiv$ minimum-size vertex-cut $\mathbf{2 3 7}$ 238
cut, nontrivial $\sim 21,33,610$
cut, odd $\sim 449,609$
minimum-capacity 449 algorithm 449
cut, $r-\sim \mathbf{8 9 6}, \mathbf{9 0 5}-907,918,974,1399$
minimum-capacity 907 min-max 907
minimum-size $905-906,918,974$ algorithm 918 min-max 905-906
cut, $S-T \sim 21,33$
cut, $s-t \sim \mathbf{2 1}, \mathbf{3 3}, \mathbf{8 7}, 131-\mathbf{1 3 2}-169$, 200-201, 974, 1020, 1413
minimum-capacity $150-156,159-$ 161, 200-201, 974, 1020, 1413 algorithm $151-156,159-160$ complexity 160-161 min-max $150-151$
minimum-size 131-169 min-max 132
planar 139-140, 161-162
minimum-capacity $161-162$

$$
\text { complexity } \quad 161-162
$$

minimum-size 139-140 complexity 139-140
cut, $S-T$ vertex-~ 22, $\mathbf{3 4}$
cut, $s-t$ vertex- $\sim \mathbf{2 2}, \mathbf{3 3}, \mathbf{1 3 2}$
minimum-size 132 min-max 132
cut, $T$-~ 488-519, 1413, 1417-1418
minimum-capacity $498-500,507-$ 510 algorithm 499-500
minimum-size $499,507-508,1413$ min-max $499,507-508$
cut, tight $\sim 609,619$
cut, trivial ~619
cut, vertex-~ 22, 33, 239-241, 243, 253
minimum-size 239-241 algorithm 239-241 complexity 241
cut arc 33
cut condition 1227-1230, 1321, 1419
$\sim$ for digraphs 1227-1228
cut condition, homotopic $\sim 1366$
cut cone $1342-\mathbf{1 3 4 3}-1345,1350,1459$
facets 1350
cut covers, disjoint directed $\sim$ 962-968
min-max $967-968$
source-sink connected 966-967
algorithm 967
min-max $966-967$
cut function $\mathbf{7 6 9}$
cut polytope $1342-1344,1348-1350$
facets 1350
cut polytope, $r-\sim \mathbf{9 0 7}$
cut polytope, $s-t \sim 199,203$
adjacency 203
vertices 203
cut polytope, $T$-~ 498-499, 507-510
cut problem, all-pairs minimum-size $\sim$ 248
cut vertex $\mathbf{2 2}$
cuts, covering by directed $\sim 218$
acyclic 218
$\min -\max 218$
cuts, disjoint $\sim 960,1030-1031,1236-$
1237, 1257-1261, 1276-1278,
1304-1305, 1309, 1313, 1316,
1320, 1354, 1414
2-commodity 1257-1261
cuts, disjoint $D_{0-\sim}$ 971-973
min-max 271-972
cuts, disjoint directed ~ 947-949, 954956, 960, 972, 1020, 1024
algorithm 954-956
min-max 247-949
cuts, disjoint $r-\sim$ 896-897, 972, 1024
$\min -\max 896$
cuts, disjoint $s-t \sim$ 87-88, 96-97, $126,1026,1413$
min-max $88,96-97$
cuts, disjoint $T-\sim$ 488-490, 501-507, $518,1413,1417-1418$
complexity 518
min-max 489-490
cuts, union of directed $\sim 224-226$
acyclic 224-226
cuts, union of disjoint $s-t \sim 211-212$
algorithm 212
$\min -\max \quad 211-212$
cutting plane 84,984
cycle $\mathbf{6 4 5}$
in graph 20
of binary hypergraph 1406,1409
of binary matroid 1424
cycle, directed ~ $\mathbf{3 2}$
cycle, $k$-~ 1409
cycle-cancelling 179-181
cycle in binary matroid 655
cycle matroid 657
cycle of binary hypergraph, even $\sim$ 1406, 1409
cycle of binary hypergraph, odd $\sim$ 1406
cycle polytope of binary matroid 1424-1425
adjacency 1425
facets 1425
cycling matroid, $1-\sim 1421,1461$
cycling matroid, 2-~ 1421-1422
cycling matroid, $3-\sim 1422-1423$
cycling matroid, 4-~ 1423-1424
cycling matroid, $\infty$ - $\sim 1420,1423$ 1424
cycling matroid, $k$-~ $\mathbf{1 4 2 0}$
$D_{0}$-cut 970-976
minimum-capacity 974 $\min -\max 974$
minimum-size 973-976
min-max $973-974$
$D_{0}$-cuts, disjoint $\sim 971-973$
min-max $971-972$
dart 1121
dart-free graph 1121
decision problem 40
decomposing edges into closed curves 1354
decomposition, brick ~ 612-613
decomposition, ear-~ 93, 252-253, 425-427, 511, 647
decomposition, odd ear- $\sim 425$
decomposition, proper ear-~ 252
decomposition, straight $\sim 1355$
decomposition property, integer $\sim \mathbf{8 2}-$ 83, 204
decomposition theorem, Dilworth's $\sim$ 218-220, 232, 235-236, 1137
defect form of Hall's marriage theorem 380-381
degree
of vertex of graph $\quad \mathbf{1 7}$
of vertex of hypergraph 1380
degree, maximum $\sim$
of graph 17
of hypergraph 1380
degree of graph, minimum $\sim \mathbf{1 7}$
degree of vertex of graph, total $\sim \mathbf{5 1 8}$
degree-sequence $\mathbf{5 7 3}$
degree-sequence of vector $\mathbf{5 6 8}$
degrees, subgraph with prescribed $\sim$ 586
deleting arc of digraph 30
deleting arc set of digraph $\mathbf{3 0}$
deleting edge
in pair $G, T \quad 504$
of graph 18
of hypergraph 1376
of signed graph 1202, 1330
deleting edge set of graph 18
deleting element of matroid 653
deleting vertex
of digraph 30
of graph 18
of hypergraph 1376
of signed graph 1202,1330
deleting vertex set
of digraph 30
of graph 18
deltoid 660-661
demand 13
demand digraph 1221
demand function 13
demand graph 1222
dependent set in matroid 651, 746
depth-first search 89
depth-first search tree $\mathbf{8 9}$
deshrinking 453
detachment, $b$-~ $\mathbf{7 0 4}$
determined by, polyhedron $\sim \mathbf{6 0}$
diameter
of graph 19
of polytope 65
diameter, monotonic $\sim$
of polytope 990
diamond 1121
diamond-free graph 1121
digraph $\equiv$ directed graph 28
Dijkstra's method 97-101, 126-128
Dilworth truncation 820-821-825
Dilworth's decomposition theorem
218-220, 232, 235-236, 1137
diperfect digraph 1131-1132
diperfect digraph, $\alpha-\sim$ 1131, 1458
diperfect digraph, $\chi$-~ 1132
directed 1-tree $\mathbf{9 9 3}$
shortest 993
directed Chinese postman problem

## 192, 518

directed circuit 32
directed circuit, shortest $\sim 94$
directed circuits, disjoint $\sim$ 958-959, 1368
complexity 959
planar 958 min-max 958
directed cut 33, 116, 218-220, 946968, 972, 1020, 1024, 1399
acyclic 219-220
maximum-size 219-220
min-max 219-220
minimum-capacity 966-967
minimum-mean capacity 968
minimum-size 962-968
min-max 967-968
source-sink connected 966-967
minimum-capacity 966-967
min-max 966-967
minimum-size 966
min-max 966
directed cut cover 946-968, 972, 1020, 1024, 1399
minimum-size 947-949, 953-954, 956, 960, 972, 1020, 1024
algorithm 953-954
complexity 956
min-max 947-948
minimum-weight 948-949, 953-
954, 956, 972, 1020, 1024
algorithm 953-954
complexity 956 min-max 948-949
directed cut cover polytope 949-950
directed cut covers, disjoint ~ 962-968
min-max 967-968
source-sink connected 966-967
algorithm 967
min-max 266-967
directed cut $k$-cover 950-951, 953954, 964-966, 968
minimum-size 950-951, 953-954
algorithm 953-954
min-max 950-951
minimum-weight 950, 953-954
algorithm 953-954
min-max 950
directed cuts, covering by $\sim 218$
acyclic 218
min-max 218
directed cuts, disjoint $\sim$ 947-949, 954956, 960, 972, 1020, 1024
algorithm 954-956
min-max 947-949
directed cuts, union of $\sim 224-226$ acyclic 224-226
directed cycle $\mathbf{3 2}$
directed edge $\mathbf{2 8}$
directed edge of bidirected graph $\mathbf{5 9 4}$, 1201
directed forest $\mathbf{3 4}$
directed graph $\mathbf{2 8}$
directed Hamiltonian circuit 115, 981
NP-completeness 115
directed Hamiltonian path problem 114
NP-completeness 114
directed path 31, 218 acyclic 218
maximum-size 218 $\min -\max 218$
directed tree $\mathbf{3 4}$
directed walk $\mathbf{3 1}$
directed walk, closed $\sim \mathbf{3 2}$
directed walk, Eulerian ~ $\mathbf{3 4}$
disconnect 21
disconnecting arc set $\mathbf{3 3}$
disconnecting arc set, $S-T \sim \mathbf{3 3}$
disconnecting arc set, $s-t \sim \mathbf{3 3}$
disconnecting edge set $\mathbf{2 1}$
disconnecting edge set, $S-T \sim \mathbf{2 1}$
disconnecting edge set, $s-t \sim \mathbf{2 1}$
disconnecting vertex set $\mathbf{2 2}, 33$
disconnecting vertex set, $S-T \sim \mathbf{2 2}$,

$$
\mathbf{3 4}, \mathbf{1 3 1}-132
$$

minimum-size $131-132$ min-max $131-132$
disconnecting vertex set, $s-t \sim \equiv$ $s-t$ vertex-cut 22, $\mathbf{3 3}$
disconnects sets, vertex set $\sim \mathbf{2 2}, \mathbf{3 4}$
disconnects vertices, vertex set $\sim \mathbf{2 2}$, 33
discrepancy of digraph 1204
discrete sandwich theorem, Frank's ~ 799
disjoint 10
disjoint arborescences $905,908,923$ 926
min-max 905,908
disjoint arborescences theorem, Edmonds' ~ 905, 974, 1047, 1399
disjoint arcs 29
disjoint bases 727, 732, 734, 736
algorithm 732, 734, 736
min-max 727
disjoint bibranchings theorem 941943, 974
disjoint branchings 904-905, 922
characterization 904-905
complexity 922
disjoint branchings theorem, Edmonds'

$$
\sim 904
$$

disjoint common bases $740-741$
$\min -\max \quad 740$
disjoint common spanning sets 741 $\min -\max \quad 741$
disjoint common transversals 402-405
min-max 402-403
disjoint connectors 877-880, 888-889
algorithm 879-880, 888-889
min-max 877-878
disjoint cuts 960, 1030-1031, 12361237, 1257-1261, 1276-1278, 1304-1305, 1309, 1313, 1316, 1320, 1354, 1414
2-commodity 1257-1261
disjoint $D_{0}$-cuts 971-973
min-max $971-972$
disjoint directed circuits $958-959,1368$
complexity 959
planar 958
min-max 958
disjoint directed cut covers 962-968
min-max $967-968$
source-sink connected 966-967
algorithm 967
min-max $966-967$
disjoint directed cuts 947-949, 954956, 960, 972, 1020, 1024
algorithm 954-956
min-max 947-949
disjoint edge covers $324-325,478-479$, 974
bipartite $324-325,974$
min-max $324-325$
disjoint edge covers, union of $\sim 350$
bipartite 350
min-max 350
disjoint edges $\mathbf{1 7}$
disjoint forests 892
disjoint homotopic paths problem

## 1368

disjoint maximal chains 235
min-max 235
disjoint odd circuits 1335-1340
min-max 1335-1340
disjoint on, path ~ 140
disjoint paths $1223-1225,1228,1233-$ 1234, 1239, 1242-1245, 1248, 1251, 1254, 1261-1265, 1267,
1271-1273, 1279-1296, 1298-
1300, 1303-1304, 1307-1311,
1313, 1315-1316, 1318, 1320-
1325, 1352, 1361, 1366-1371, 1458-1459
complexity 1224-1225, 1243-1244, 1273, 1309, 1323, 1366, 1459
directed 1223-1225, 1243-1245, 1262-1263, 1289, 1309, 1322, 1368-1370
NP-completeness 1234
planar 1299
complexity 1299
disjoint paths, arc-~ 132, 906, 1307
disjoint paths, edge-~ 1253, 1255,
1285, 1296-1299, 1308, 1311-
1313, 1318-1320
planar 1296-1299, 1308, 13111313, 1318-1320
algorithm 1298
characterization 1296-1298, 1308, 1311-1313, 13181320
complexity 1299
disjoint paths, internally vertex-~ $\mathbf{1 3 2}$
disjoint paths, openly $\sim \equiv$ internally vertex-disjoint paths
disjoint paths, vertex-~ 1224-1225, 1243, 1299, 1320-1323, 13681370
complexity 1224-1225, 1243
planar 1299, 1320-1323, 1368-1370
algorithm 1320-1323
characterization 1320-1323 complexity 1299
disjoint paths problem 1223
fractional solution 1223
half-integer solution 1223
disjoint paths problem, arc-~ 1223
disjoint paths problem, edge- $\sim 1223$
disjoint paths problem, homotopic edge-~ 1366
disjoint paths problem, $k \sim 1223$
disjoint paths problem, $k$ arc-~ 1223
disjoint paths problem, $k$ edge-~ $\mathbf{1 2 2 3}$
disjoint paths problem, $k$ vertex-~ 1223
disjoint paths problem, vertex-~ 1223
disjoint perfect matchings 326-328, 340
bipartite 326-328, 340 min-max 327
disjoint $r$-arborescences 905-907, 918922, 925, 974, 1078-1079
algorithm 918-921
complexity 921-922
min-max 905-907
disjoint $r$-arborescences, capacitated $\sim$ 922
complexity 922
disjoint $r$-cuts 896-897, 972, 1024
min-max 896
disjoint $R-S$ bibranchings 940-944, 974
min-max 941-942
disjoint $R-S$-bibranchings 942
algorithm 942
disjoint $R-S$ biconnectors 931-934, 944
algorithm 933
min-max 933
disjoint $R-S$ bicuts 937, 972, 1024
$\min -\max 937$
disjoint $\mathcal{S}$-paths 1280-1281
min-max 1280-1281
disjoint $\mathcal{S}$-paths, vertex-~ $1280-1281$
min-max 1280-1281
disjoint $\mathcal{S}$-paths theorem, Mader's $\sim$ 1280-1281
disjoint $s-t$ cuts 87-88, 96-97, 126, 1026, 1413
min-max 88, 96-97
disjoint $s-t$ cuts, union of $\sim 211-212$
algorithm 212
min-max 211-212
disjoint $S-T$ paths $131-132,140-147$
exchange properties $140-141$
min-max 131-132
disjoint $s-t$ paths, arc-~ 132, 134140, 142-147, 151
algorithm 134-138
complexity 138-139
min-max 132
planar 139-140 complexity 139-140
disjoint $s-t$ paths, edge- $\sim 139,254$, 974, 1413
planar 139 complexity 139
disjoint $s-t$ paths, internally $\sim 132$, 137-140, 142-147, 275-276
algorithm $137-138$
complexity 139,276
min-max 132
planar 140 complexity 140
disjoint $s-t$ paths, internally vertex-~ 132, 137-140, 142-147, 275-276
algorithm 137-138
complexity 139,276
min-max 132
planar 140 complexity 140
disjoint spanning trees $877-880,888-$ 892, 1456
algorithm 879-880, 888-889
complexity 889-890
fractional 891 complexity 891
min-max $877-878$
disjoint strong connectors 973-976
algorithm 975-976
min-max 973-974
disjoint subgraphs 18, 30
disjoint subgraphs, arc-~ 30
disjoint subgraphs, edge-~ 18
disjoint subgraphs, vertex-~ 18, 30
disjoint $T$-cuts $488-490,501-507,518$, 1413, 1417-1418
complexity 518
min-max $489-490$
disjoint $T$-joins $\quad 507-510,519,1413$, 1456
min-max 507-508
disjoint $T$-paths 1279-1295
algorithm 1283-1284
min-max 1279-1280
disjoint $T$-paths, edge-~ 1282-1283, 1285-1286
algorithm 1285-1286
min-max 1282-1283
disjoint $T$-paths, internally $\sim 1282$ $\min -\max 1282$
disjoint $T$-paths, internally vertex-~ 1282
$\min -\max 1282$
disjoint $T$-paths, vertex-~ 1279-1280, 1283-1284
algorithm 1283-1284
min-max 1279-1280
disjoint $T$-paths theorem, Gallai's ~ 1279-1280
disjoint $T$-paths theorem, Mader's edge-~ 1282-1283, 1289
disjoint $T$-paths theorem, Mader's internally ~ $\mathbf{1 2 8 2}$
disjoint transversals 385-386, 728
min-max 385,728
disjoint trees $1242,1322,1325,1371$
complexity 1325
planar 1242 algorithm 1242
disjoint trees problem, vertex-~ 1242, 1322
disjoint trees theorem, Tutte-Nash-Williams'~ 877878, 931, 1048
disjoint walks 20, 32
disjoint walks, arc-~ 32
disjoint walks, edge-~ 20
disjoint walks, internally $\sim 20,32$
disjoint walks, internally vertex- $\sim \mathbf{2 0}$, 32
disjoint walks, vertex-~ 20, 32
distance $\mathbf{1 9}, \mathbf{3 1}, \mathbf{8 7}, \mathbf{9 6}, 1226-1227$, 1237-1238, 1257-1260, 1276, $1278,1295,1304-1306,1308-$ 1309, 1313, 1317, 1320
distance, tentative $\sim 97$
distinct representatives, system of $\sim \equiv$ transversal
distributive lattice 233-235, 1034
dominant 66
dominating set $\mathbf{1 1 5 0}$
double cover, circuit ~ $\mathbf{1 4 2 7}$
double cover conjecture, circuit $\sim$ 497, 645-646, 1427, 1456
doubly linked list 48-49
doubly odd closed curve 1367
doubly stochastic matrix 302-303, 314
down hull 59
down-monotone ideal 11
down-monotone in $\mathbb{R}_{+}^{n} \mathbf{6 6}$
down-monotone subset of $\mathbb{R}^{n} \quad 65$
dual
of linear programming problem 63
of matroid 652
of planar digraph 35
of planar graph 27-28
of polymatroid $\mathbf{7 8 2}$
of submodular function $\mathbf{7 8 2}$
dual greedy algorithm 859-860
dual hypergraph 1375
dual lattice $\mathbf{8 1}$
dual matroid 652-653
dual problem 63
dual solution 63, 71
dual transportation polyhedron $\mathbf{3 4 7}$
diameter 347
dimension 347
vertices 347
duality 62-63
duality, weak ~ $\mathbf{6 2}$
duality equation, linear programming $\sim$ 63
duality theorem of linear programming 62-63
duplicating vertex 1109
of hypergraph 1376
dyadic hypergraph 1401
dynamic flow 192-195
ear 93, 252, 511
ear, adding ~ 93, 252, 511
ear, odd $\sim 425$
ear, proper $\sim \mathbf{2 5 2}$
ear-decomposition 93, 252-253, 425427, 511, 647
ear-decomposition, odd $\sim 425$
ear-decomposition, proper $\sim \mathbf{2 5 2}$
edge
of graph 16
of hypergraph 36, 1375
of polyhedron 65
edge, directed $\sim \mathbf{2 8}$
edge-colour 465
edge-colourable graph, $k$-~ 24, 465
edge-colourable graph, $k$-list-~ 335
edge-colouring 23, 321-331, 333-336, 465-484, 1016, 1136, 1455
bipartite 321-331, 333-336, 1016, 1136
algorithm 322-323, 333-334
complexity 334-335
min-max 321-322
complexity 466-467
history 482-484
NP-completeness 468-470
edge-colouring, fractional $\sim$ 474-478, 1455
complexity $477-478$
min-max 474-475
edge-colouring, $k$-~ 321, 465
edge-colouring, list-~ 335-336, 1455
bipartite 335-336
edge-colouring, minimum $\sim \mathbf{2 4}$
edge-colouring number 23, 321, 465
edge-colouring number, fractional $\sim$ 474
edge-colouring number, list-~ 335, 482
edge-colouring theorem, Kőnig's ~ 321-322, 324-325, 331, 934, 1016, 1136, 1441
edge-connected component, 2-~ 247248
algorithm 248
edge-connected component, $k$-~ $\mathbf{2 4 8}$
edge-connected graph, $k$-~ 238, 1050
minimum-size 1050
edge-connected graph, $r$-~ 1055,

## 1067

edge-connectivity 237-238, 244-251, 253-255, 1037-1040, 1044-1046, 1048, 1050, 1055-1057, 10621074, 1458
algorithm 244-246
complexity 246-247
edge-connector, 2-~ 1062-1063
minimum-size 1062-1063
formula 1062-1063
edge-connector, $k-\sim \mathbf{1 0 6 2}$, 1065-1066
minimum-size 1065-1066
algorithm 1065
min-max 1065-1066
edge-connector, $r-\sim 1067$
edge cover 23, 315-320, 461-464, 536539, 972, 1023, 1095, 1135
bipartite $316-320,1023,1135$
history 319-320
minimum-size $316-317,1023$, 1135
algorithm 316
min-max 317
minimum-weight 317-318
algorithm 317
min-max 318
in hypergraph 1428
minimum-size 315-316, 461-462,
464, 536-539, 972, 1095, 1135
algorithm 461-462
bipartite 972
min-max 461
minimum-weight $317,462-464$ algorithm 317, 462 min-max $\quad 462-464$
nonbipartite 464 history 464
edge cover, $2-\sim \mathbf{5 3 1}-532,534$
minimum-size 531-532
algorithm 532
$\min -\max 532$
minimum-weight 534 $\min -\max 534$
edge cover, $b-\sim \quad 347-354,361, \mathbf{5 7 5}-583$
bipartite 347-354, 361
minimum-size $348,352,361$
algorithm 352, 361
$\min -\max 348$
minimum-weight $348,352-353$
algorithm 352-353
min-max 348
minimum-size $351-352,575-576$, 578
algorithm 576
min-max 576,578
minimum-weight 577-578
algorithm 577-578 min-max $577-578$
edge cover, capacitated $b-\sim$ 350-353, 579-580, 583
bipartite 350-353 minimum-size 350-351 min-max $350-351$ minimum-weight 351-353 algorithm 351-353 min-max 351
minimum-size 579-580, 583 algorithm 580 min-max 579-580
minimum-weight 580 algorithm 580 min-max 580
edge cover, fractional $\sim 532-\mathbf{5 3 3}$, 1090
in hypergraph 1429
edge cover, $k$-~ $\mathbf{5 7 8}-579$
in hypergraph 1429
minimum-size 579 min-max 579
edge cover, simple $2-\sim$ 535-536
minimum-size 535-536 algorithm 535 min-max 535
minimum-weight 535-536 algorithm 536
edge cover, simple $b-\sim 349-354,581-$ 582
bipartite 349-354 minimum-size 349-350 min-max 349 minimum-weight 350-353 algorithm 350-353 min-max 350
minimum-size $581-582$ algorithm 581-582 min-max $581-582$
minimum-weight 581 min-max 581
edge cover, simple $k$-~ $\mathbf{5 8 2}$
minimum-size 582 $\min -\max 582$
edge cover number 23, 315-317, 461
edge cover number, fractional $\sim \mathbf{5 3 3}$, 1090
edge cover packing number 324, 479
edge cover polyhedron, 2-~ 533-534 edge cover polyhedron, $b-\sim \mathbf{3 4 8}, 576$ 577
bipartite 348
edge cover polyhedron, fractional $\sim$ 533
edge cover polytope $\quad 318-319,462-464$
adjacency 464
bipartite 318-319
diameter 464
edge cover polytope, $c$-capacitated $b$ - $\sim$ 580
edge cover polytope, simple $2-\sim \mathbf{5 3 6}$
edge cover polytope, simple $b-\sim \mathbf{3 5 0}$, 581
bipartite 350
edge cover theorem, Kőnig-Rado ~ 317-320, 392, 703, 960, 972, 1023, 1135-1136, 1441
edge covers, disjoint $\sim 324-325,478-$ 479, 974
bipartite 324-325, 974
min-max $324-325$
edge covers, union of disjoint $\sim 350$
bipartite 350
$\min -\max 350$
edge-disjoint paths $1253,1255,1285$, 1296-1299, 1308, 1311-1313, 1318-1320
planar 1296-1299, 1308, 13111313, 1318-1320
algorithm 1298
characterization 1296-1298, $1308,1311-1313,1318-$ 1320
complexity 1299
edge-disjoint paths problem $\mathbf{1 2 2 3}$
edge-disjoint paths problem, homotopic $\sim 1366$
edge-disjoint paths problem, $k \sim \mathbf{1 2 2 3}$ edge-disjoint $s-t$ paths 139, 254, 974, 1413
planar 139
complexity 139
edge-disjoint subgraphs 18
edge-disjoint $T$-paths 1282-1283, 1285-1286
algorithm 1285-1286
min-max 1282-1283
edge-disjoint $T$-paths theorem, Mader's ~ 1282-1283, 1289
edge-disjoint walks 20
edge inequalities $\mathbf{1 0 9 0}$
edge of graph, multiple $\sim 16$
edges, disjoint $\sim \mathbf{1 7}$
edges of graph, parallel $\sim \mathbf{1 6}$
Edmonds-Gallai decomposition 423425, 518-519, 545, 574, 765
Edmonds-Giles graph 1148
Edmonds-Giles theorem 1019-1021, 1028-1030, 1034
Edmonds graph 1211
Edmonds-Johnson property 608
Edmonds' disjoint arborescences theorem 905, 974, 1047, 1399
Edmonds' disjoint branchings theorem 904
Edmonds' matching polytope theorem 440, 442-443
Edmonds' perfect matching polytope theorem 438-439
efficient algorithm $\mathbf{3 9}$
Egerváry's theorem 285-286, 304, 318
elementary arithmetic operations $\mathbf{3 9}$
ellipsoid method 68-71
embedded graph, cellularly $\sim 1357$
embedding of graph $\mathbf{2 5}$
end
of arc 29
of directed walk $\mathbf{3 1}$
of edge $\mathbf{1 7}$
of walk 19
end arc of walk $\mathbf{3 1}$
end edge of walk 19
end point of curve $\mathbf{1 3 6 1}$
end vertex
of graph 17
of walk 19, 31
enter 29
entry of vector $\mathbf{1 1}$
equality, implicit $\sim \mathbf{6 4}$
equivalent graphs, $P_{4} \sim \mathbf{1 1 2 2}$
equivalent signed graph 1329
equivalent signing $\mathbf{1 3 2 9}$
essential edge 1133
Euclidean traveling salesman problem 982, 990

Euler 1420
condition in matroid 1420
Euler condition 1233-1236, 1241, 1244, 1251-1252, 1254-1255, 1262-1263, 1266-1267, 12711274, 1289, 1291-1292, 12961299, 1301-1302, 1304, 13071309, 1311-1312, 1315-1316, 1318-1320, 1324, 1341-1342, 1361, 1366-1367, 1459
in matroid $1420,1422-1423$, 1425-1426
Euler condition, global ~ 1366
Euler condition, local ~ $\mathbf{1 3 6 6}$
Euler's formula 26
Eulerian digraph 34, 952, 957-958, 1234, 1254, 1262-1263, 1289
Eulerian directed walk 34
Eulerian graph 24, 472, 488, 518, 1238-1240, 1252, 1263, 1289, 1299, 1301, 1315, 1336, 1340, 1350, 1354, 1356
Eulerian orientation 34, 91
algorithm 91
Eulerian signed graph 1335
Eulerian walk 24
even circuit 1329
in bidirected graph 1201
even component 20
even cycle of binary hypergraph 1406 , 1409
even edge set 1329
even face of planar graph 1144
even pair of vertices $\mathbf{1 1 2 4}$
even path 1329
even set $\mathbf{9}$
even walk 19
evenly bipartite graph 1340-1341
evenly bipartite signed graph 1331, 1340
characterization 1340
exact realization $\mathbf{1 0 5 1}$
exactly realizable function 1051
excess function 149, 1047
exchange properties of bases 669-671, 722, 728-729
exchange properties of branchings 909-910
exchange properties of disjoint paths 140-141
exchange properties of forests 867-868
exchange properties of independent sets 654, 669-671
exchange property, Steinitz' ~ 654, 676
exchange property of common independent sets 721-722
exchange property of common transversals 407-408
exchange property of matching forests 1008-1011
exchange property of matchings $266-$ 267
exchange property of transversals 381, 386-387
extended contrapolymatroid $\quad \mathbf{7 7 4}$
extended polymatroid $\mathbf{7 6 7}$
extension, linear ~ $\mathbf{1 1}$
extension, parallel $\sim \mathbf{7 3 9}$
extremal ray of polyhedron $\mathbf{6 5}$
extreme common independent set $\mathbf{7 0 7}$
extreme forest $\mathbf{8 6 7}$
extreme function 183-184
extreme matching 287
extreme stable set 1213
$f$-augmenting path 151
$\mathcal{F}$-contraction 610
$F$-cover 1203
$\mathcal{F}$-critical vertex set $\mathbf{5 4 5}$
$f$-flat $\mathbf{7 7 7}$
$f$-inseparable subset of polymatroid 777
$\mathcal{F}$-matching $\mathbf{5 4 5}$
$\mathcal{F}$-matching, maximum $\sim \mathbf{5 4 5}$
$\mathcal{F}$-matching, perfect $\sim \mathbf{5 4 5}$
$F$-stable set 1203
face
of embedded graph 1355
of planar graph 25
of polyhedron 63-64
face-determining inequality 64
face-inducing inequality 64
face of planar graph, bounded $\sim \mathbf{2 6}$
face of planar graph, even $\sim 1144$
face of planar graph, odd $\sim \mathbf{1 1 4 4}$
face of planar graph, unbounded $\sim$
face of polyhedron, minimal $\sim \mathbf{6 4}$ facet
of convex set 1165
of polyhedron $\mathbf{6 4 - 6 5}$
facet, rank ~ 1216
facet-determining inequality $\mathbf{6 4}$
facet-inducing inequality $\mathbf{6 4}$
facet of polyhedron 64-65
factor, $1-\sim \equiv$ perfect matching 415, 425-428, 431-436
factor, 2-~ $\mathbf{5 2 7}-528,531,545,986-$ 987, 1456
algorithm 528
characterization 527-528
complexity 545
minimum-weight 528, 531, 986-987
algorithm 528
min-max 531
factor, $b-\sim$ 340-343, 358, 569-574, 621
algorithm 572
bipartite 340-343, 358
algorithm 342-343
characterization 340
complexity 358
minimum-weight 341-343
algorithm $\quad 342-343$
min-max 341
characterization 570
minimum-weight 571-572
algorithm 572
min-max 571
factor, $k-\sim \mathbf{3 2 7}, \mathbf{3 4 0}, 572-574$
bipartite 327,340
characterization 327,340
characterization 572
factor, replicating vertex by $\sim \mathbf{1 1 0 9}$
factor-critical graph 424-425-426, 446, 544-545
factor polytope, 2-~ 530
diameter 530
facets 530
factor polytope, $b-\sim$ 570-571
factor theorem, Tutte's 1-~ 414-415, 425, 435-436
family $\mathbf{9}$
family, cross-free $\sim \mathbf{3 7}, \mathbf{2 1 4}-216, \mathbf{8 4 2}$, 1021-1022
family, crossing ~ 838-851, 976-980, 1018-1023
family, intersecting ~832-837
family, laminar $\sim \mathbf{3 7}, \mathbf{2 1 4}-215,441$,
453, 712, 820, 832
family, lattice ~ 826-832, 834-835
Fano hypergraph 1386
Fano matroid 655
Farkas' lemma 61
fattest augmenting path 159
feasible circulation $\mathbf{1 7 8}$
feasible direction $\mathbf{7 3}$
feasible multiflow 1221
feasible problem 63, 1221
feasible region 63
feasible solution 14,63
feasible spanning tree 207
feasible system of linear inequalities 61
feedback arc set 951-953, 956-958
minimum-size 951-953, 956-958
planar 951, 958
min-max 951
minimum-size 958
$\min -\max 958$
shortest 951-953
complexity 951
feedback vertex set 958-959
Fekete's lemma 14-15
Fibonacci forest 99-100
Fibonacci heap 99-100-101
finite matroid $\mathbf{7 4 6}$
finitely generated cone $\mathbf{6 0}$
first arc of walk 31
first edge of walk 19
first vertex of walk 19,31
fixed point of curve 1369
flat, $f$-~ $\mathbf{7 7 7}$
flat of matroid $\quad \mathbf{6 6 6}-668, \mathbf{6 9 8}$
flow 148-169, 172-173, 176-191, 195197, 205-207, 1020
in matroid 1426
in undirected graph 1222
maximum 1020
minimum-cost $\quad 177-191,195-197$
algorithm 185
simplex method 195
flow, 2-commodity ~ 1251-1265, 1414
characterization 1252-1254
flow, blocking ~154-156
flow, concave-cost ~196-197
flow, convex-cost ~ 196
flow, dynamic ~192-195
flow, generalized $\sim 196$
flow, maximum ~ 148-149-169, 173, 200-201, 1020, 1453
algorithm $151-160,1453$
complexity 160-161
history 164-169
min-max $150-151$
planar 161-162
complexity 161-162
simplex method 162-163
flow, maximum $s-t \sim 149$
flow, multicommodity $\sim \equiv$ multiflow
flow, nowhere-zero ~470-473, 646, 1426-1427, 1454
in matroid 1426-1427
flow, nowhere-zero $k-\sim 472$
flow, polymatroidal network $\sim 1028$ 1029
flow, $s-t \sim 148$
flow, submodular ~ 1018-1021, 1034
minimum-cost 1019-1020, 1034 algorithm 1019-1020, 1034 min-max 1019
flow, unsplittable ~ 196
flow-augmenting algorithm 151
flow-augmenting path 151
flow conjecture, $3-\sim 472,1454$
flow conjecture, 4-~ 472, 498, 645, 1426, 1454
flow conjecture, 5-~ 472, 646, 1453
flow conjecture, weak $3-\sim 473,1454$
flow conservation law 148
flow homotopic to $\mathbf{1 3 6 4}$
flow over group $\mathbf{4 7 0}$
flow polyhedron, submodular ~1018, 1034
dimension 1034
facets 1034
flow problem, maximum $\sim 149$
flow problem, minimum-cost $s-t \sim$ 177
flow with upper and lower bounds 172-173
flow with upper and lower bounds, maximum $\sim 173$
flower, $M-\sim 416$
flowing matroid, 1-~ 1421, 1461
flowing matroid, 2-~ 1421-1422
flowing matroid, 3-~ 1422-1423
flowing matroid, 4-~ 1423-1424
flowing matroid, $\infty$-~ 1420, 14231424
flowing matroid, integer $\infty$-~ 1420
flowing matroid, integer $k$ - $\sim 1420$ 1421
flowing matroid, $k$-~ 1420
Floyd-Warshall method 110-111
Ford's method 115
forest 22, 855, 860-861, 867-868
exchange properties $867-868$
in hypergraph $\mathbf{7 5 5}$
longest 860-861
algorithm 860
min-max $860-861$
forest, directed ~ $\mathbf{3 4}$
forest, $M$-alternating $\sim 420$
forest, matching ~ 1005-1017
exchange property 1008-1011
maximum-size $1006-1007,1016$ min-max 1006-1007
maximum-weight 1012-1016 min-max 1012-1016
forest, maximal ~ 855
forest, perfect matching $\sim$ 1007-1008
algorithm 1008
characterization 1007-1008
forest, rooted ~ $\mathbf{3 4}$
forest cover 869-870
forest cover polytope $\mathbf{8 7 0}$
forest-merging method $\mathbf{8 5 7}-859, \mathbf{8 7 1}$, 874
forest-merging method, parallel $\sim$ 859, 871-874
forest polytope $\mathbf{8 6 1}, 879-884,886$
forest polytope, matching $\sim 1011$ 1017
facets 1017
forests, covering by $\sim 878-879,888-$ 890
algorithm 888
complexity 889-890
min-max 879
forests, covering by matching $\sim 1016$
$\min -\max 1016$
forests, disjoint $\sim 892$
forests, union of $\sim$ 877, 890
maximum-size 890 complexity 890
maximum-weight 890 complexity 890
min-max 877
forward arc 31
four-colour conjecture 1085
four-colour theorem 26-27, 470-471, 473, 476, 482-484, 498, 1085, 1087
fractional arboricity 891
fractional $b$-matching polytope 561 vertices 561
fractional $c$-covering $\mathbf{3 7}$
fractional $c$-packing 36
fractional clique cover number 1096
fractional colouring, minimum $\sim 1096$, 1098
fractional colouring number 1096
fractional covering 36
fractional edge-colouring 474-478, 1455
complexity 477-478
min-max 474-475
fractional edge-colouring number $\mathbf{4 7 4}$
fractional edge cover 532-533, 1090
in hypergraph 1429
fractional edge cover number 533, 1090
fractional edge cover polyhedron 533
fractional matching 521, 1094
in hypergraph 1378
fractional matching number 521, 1094
fractional matching polytope 522
fractional multiflow 1222, 1224-1231, 1234-1239, 1241, 1245-1249, 1270, 1272-1274, 1287, 13071308, 1317-1318, 1320, 13411342, 1354, 1357, 1361, 1368, 1459
algorithm 1225-1226
maximum-value 1226-1227
fractional packing $\mathbf{3 6}$
fractional solution of disjoint paths problem 1223
fractional stable set $\quad 532-533,1090-$ 1093, 1095-1096, 1099
in hypergraph 1429
maximum-weight 1091 algorithm 1091
fractional stable set, strong ~1096, 1098-1099
maximum-size 1096
fractional stable set number 533, 1090
fractional stable set number, strong $\sim$ 1096
fractional stable set polytope 10901093
vertices 1091-1092
fractional vertex-colouring, minimum $\sim$ 1096, 1098
fractional vertex cover 521, 10931095
in hypergraph 1378, 1380-1381 minimum-size 1380-1381
minimum-weight 1094 algorithm 1094
fractional vertex cover number 521, 1093
fractional vertex cover polytope 10941095
vertices 1094
fractional weighted clique cover number 1097
fractional weighted colouring, minimum ~ 1097
NP-completeness 1097
fractional weighted colouring number 1097
Frank's discrete sandwich theorem $\mathbf{7 9 9}$
freely homotopic closed curves 1352
freely homotopic to, circulation $\sim$ 1357, 1360
Frobenius' theorem 261-263, 276-277, 280
Fulkerson conjecture 476, 1455
Fulkerson conjecture, generalized $\sim$ 476, 509-510, 645, 1454
Fulkersonian hypergraph 1383
function, extreme ~ 183-184
function, integer $\sim 11$
function, modular $\sim \mathbf{7 6 6}$
function, nondecreasing $\sim \mathbf{7 6 6}$
function, nonincreasing $\sim \mathbf{7 6 6}$
function, submodular $\sim$ 665, 766-826-852, 1018-1034
operations on 781-782
function, supermodular $\sim \mathbf{7 6 6}, 774-$ 775, 1022-1023
function, symmetric ~ 1051
fundamental circuit in matroid matching $\mathbf{7 4 7}$
fundamental cut 449, 499
Gale-Ryser theorem 359-361
Gale-Shapley theorem 311-312, 335
Gale's theorem 174
Gallai graph 1143, 1145
Gallai-Milgram theorem 232-233, 1453
Gallai's disjoint $T$-paths theorem 1279-1280
Gallai's theorem 316
$\Gamma$-free matrix 1445-1446
$\Gamma$-metric 1273, 1316
$\Gamma$-metric condition 1273, 1316
$\gamma$-pluperfect graph $\mathbf{1 1 8 2}$
gammoid 659-661, 739, 765
gammoid, strict ~ 659-661
generalized flow 196
generalized Fulkerson conjecture 476, 509-510, 645, 1454
generalized matroid 852
generalized polymatroid 845-849, 1020-1021
generalized polymatroid, dimension of $\sim 849$
generalized polymatroid intersection 847-849
generalized submodular function $\mathbf{8 5 1}$
generalized switchbox 1324
generated by, cone $\sim \mathbf{6 0}$
generated by tree and digraph, network matrix ~ 213
generates a collection, collection $\sim$ 1032
geometric lattice $\mathbf{6 6 8}$
global Euler condition 1366
Gomory-Hu tree 248-253
algorithm 250-251
complexity 251
Gomory-Hu tree for vertex set $\mathbf{2 5 0}$
good algorithm 39
good characterization 42-43
good collection 1074
good connector 859
good forest $\mathbf{8 5 6}, \mathbf{8 6 6}, 868$
good pair 1074
gradient method 73
graph 16
graph, bidirected ~ 594-608, 12011203
graph, directed $\sim \mathbf{2 8}$
graph, equivalent signed $\sim 1329$
graph, $k$-~ 475, 644-645, 1454
graph, mixed ~ 30, 926, 1005-1017, 1037-1038, 1048, 1062, 1074
graph, signed ~ 1329
graph, topological ~ 25
graph, undirected ~ $\mathbf{1 6}$
graphic matroid 657, 754-755, 823
greedy algorithm 688-690, 699, 771773, 856-859
greedy algorithm, dual ~ 859-860
grid 1323-1325
grid, rectangular ~ 1323
grid graph 1323
group, flow over $\sim 470$
Guenin's theorem 1329-1340-1341, 1392-1394
Győri's theorem 1032-1034, 11001101
$H$ minor 25
h-perfect graph 1207
$H$-subdivision 25
$H$ subgraph 18
Hadwiger's conjecture 1086-1087, 1457
Hajós' conjecture 1087-1088
half-integer 2-commodity flow 12511256
algorithm 1254
half-integer multiflow 1222, 12301231, 1234, 1236, 1238, 1251, 1253-1255, 1258, 1266, 12711274, 1288, 1290-1291, 1294, 1298, 1310, 1318, 1341-1342, 1361, 1459
complexity $1231,1234,1273,1310$
half-integer multiflow problem 1222
half-integer solution of disjoint paths problem 1223
half-integer vector $\mathbf{7 9}$
half-integral, totally dual $\sim 81$
halfspace, affine ~59, 607
halfspace, linear ~ 59
halfspace, rational affine $\sim 607$
Hall's condition $\mathbf{3 7 9}$
Hall's marriage theorem 379-380, 392
Hall's marriage theorem, defect form of ~ 380-381
Hall's theorem 379-380, 392
Hamiltonian circuit 24, 34, 981-982, 996
longest 996
shortest 981-982
Hamiltonian circuit, directed $\sim 115$, 981
NP-completeness 115
Hamiltonian circuit, undirected $\sim 115$
NP-completeness 115
Hamiltonian digraph $\mathbf{3 4}$
Hamiltonian graph 24
Hamiltonian path $24,34,114$
Hamiltonian path problem, directed $\sim$ 114
NP-completeness 114
Hamiltonian path problem, undirected ~ 114-115
NP-completeness 114-115
Hamming distance 1173
handle 987
have colour 321
head of arc $\mathbf{2 9}$
heap $98-99,128-129$
heap, 2-~ 98-99, 128-129
heap, Fibonacci ~ 99-100-101
heap, $k$-~ 98-99, 128-129
height
of element of partially ordered set
$217,312,429,1137$
Hilbert base 81-82
Hilbert base, integer $\sim 81$
Hirsch conjecture 65, $\mathbf{1 4 5 3}$
history of assignment problem 292-300
history of bipartite edge cover 319-320
history of bipartite matching 278-284
history of Chinese postman problem 519
history of edge-colouring 482-484
history of edge cover 319-320, 464
bipartite 319-320
history of efficiency and complexity 49-58
history of machine configuration 45
history of matroid union 743-744
history of matroids 672-687
history of maximum flow 164-169
history of Menger's theorem 142-147
history of multiflow 1249-1250
history of nonbipartite matching 431437
history of perfect graphs 1176-1185
history of polyhedral combinatorics 67
history of shortest path 119-130
history of shortest spanning tree 871876
history of transportation 362-377
history of transshipment $362-377$
history of transversals 390-392
history of traveling salesman problem 996-1004
history of weighted bipartite matching 292-300
Hitchcock-Koopmans transportation problem $\mathbf{3 4 4}$
Hitchcock's theorem 344-345
Hoffman's circulation theorem 171172, 1020
hole $1085,1107,1366$
hole, anti~ 1085, 1107
hole, odd ~1085, 1107
hole, odd anti~ 1085, 1107
homeomorph $\mathbf{2 5}$
homeomorphic graphs 25
homogeneous pair 1112
homomorphism 1207
homotopic 1362
homotopic circulation theorem 13571360
homotopic closed curves, freely $\sim$ 1352
homotopic cut condition 1366
homotopic edge-disjoint paths problem 1366
homotopic paths problem, disjoint $\sim$ 1368
homotopic to, circulation freely $\sim$ 1357, 1360
homotopic to, flow $\sim \mathbf{1 3 6 4}$
homotopy 1352-1371
horizontally convex polyomino 1149
Hu's 2-commodity flow theorem 1253-1254
hull, convex $\sim 59$
hull, down $\sim 59$
hull, integer ~ 83-84, 607, 1098
hull, up ~ 59
Hungarian method 286-290, 294, 298300, 305-307
hypergraph 36, 755, 1375-1451
hypergraph, blocking ~ 1377
hypergraph, connected $\sim \mathbf{3 6}$
hypergraph, contracting vertex of $\sim$ 1376
hypergraph, dual ~ 1375
hypergraph, $k$-uniform $\sim \mathbf{3 6 , ~} \mathbf{7 5 5}$
hypergraph, parallelization of $\sim \mathbf{1 3 7 6}$
hypergraph, partial ~ 1439
hypermetric cone 1345
hypermetric inequalities 1345
hyperplane 12
hyperplane, supporting $\sim 63$
ideal 233
ideal, lower ~ 11, 233, 1026, 1028
ideal, upper $\sim 11,1028$
ideal hypergraph 1383-1396, 14601461
ideal hypergraph, binary $\sim$ 1408-1409, 1460-1461
ideal matrix 1396
ILP $\equiv$ integer linear programming
image 25, 417
imperfect graph, critically $\sim \equiv$ minimally imperfect graph 1107
imperfect graph, minimally ~ 11071109, 1113, 1115-1125, 1145, 1150
implicit equality $\mathbf{6 4}$
improve a collection of matroid matchings $\mathbf{7 5 7}$
incidence matrix
of bidirected graph 594, 1201
of digraph 35, 204
of family of sets 12
of graph 28
of hypergraph 1375
incidence vector 11
incident 17, 25, 29
incident with edge, set $\sim \mathbf{1 7}$
inclusionwise maximal 10
inclusionwise minimal 10
indegree of vertex 29
independence testing oracle $\mathbf{6 8 9}$
independent collection of pairs of subsets 1032
independent path-matching $\mathbf{7 6 4}$
independent path-matching vector $\mathbf{7 6 4}$
independent set 651, 654, 669-671, 688-692, 746
exchange properties 654, 669-671
maximum-weight 688-692
algorithm 688-690
min-max 690-691
independent set, common $\sim \mathbf{7 0 0}-701$, 705-724, 768, 1026
exchange property $721-722$
maximum-size 700-701, 705-707, 710, 1026
algorithm 705-707
complexity 707, 710 min-max $700-701$
maximum-weight 707-712, 714-715 algorithm 707-712 min-max 714-715
of three matroids 700,707 NP-completeness 700, 707
independent set augmenting algorithm, common ~ 705-706
independent set in graph $\equiv$ stable set
independent set polytope 690-699, 730-731, 733
adjacency 698-699
facets 698
independent set polytope, common $\sim$ 712-714-719, 741-743
facets 718-719
independent sets, covering by $\sim 726-$ 727, 729, 732, 735-736
algorithm 732, 735-736
min-max 727, 729
independent sets, covering by common ~ 739-740
$\min -\max \quad 740$
independent sets, union of $\sim 726$
matroid union theorem 726

## min-max 726

independent transversal 702
characterization 702
independent vectors, affinely $\sim 13$
independent vectors, linearly $\sim 13$
induced by, subgraph $\sim 18,30$
induced subgraph 18, 30
induction of matroid 736-737
induction of polymatroid 782-783
inequality, active $\sim 63$
inequality, facet-determining $\sim 64$
inequality, facet-inducing $\sim 64$
inequality, tight $\sim 63$
inequality, valid $\sim \mathbf{6 0}$
inequality problem, most violated $\sim$ 697-698, 733
infeasible problem 63
infinite matroid 745
injection 13
injective function $\mathbf{1 3}$
inneighbour 29
input of problem 40
input size 39
input size of vector 69
inseparable subset of matroid $\mathbf{6 9 8}$
inseparable subset of polymatroid, $f-\sim$ 777
instance of problem 40
integer $\infty$-flowing matroid 1420
integer decomposition property 82-83, 204
integer function 11
integer Hilbert base $\mathbf{8 1}$
integer hull 83-84, 607, 1098
integer $k$-commodity flow problem 1222
integer $k$-flowing matroid $\quad 1420-1421$
integer linear programming 73-74-84
integer multiflow 1222-1225, 12301231, 1234-1235, 1239-1241, 1251, 1254-1255, 1257, 1266, 1271-1274, 1286-1288, 12901292, 1307, 1318, 1320, 1334, 1342
complexity $1224-1225,1231,1251$
integer multiflow, half-~ 1222, 12301231, 1234, 1236, 1238, 1251, 1253-1255, 1258, 1266, 12711274, 1288, 1290-1291, 1294,

1298, 1310, 1318, 1341-1342, 1361, 1459
complexity $1231,1234,1273,1310$
integer multiflow, quarter-~ 1231, 1233-1234, 1236, 1274, 1318
integer multiflow problem 1222
integer polyhedron 74-81
integer polyhedron, box-~ $\mathbf{7 5}, 1418$
integer rounding property 82-83
integer vector 11, 73
integer vector, half- $\sim \mathbf{7 9}$
integrality, primal $\sim \mathbf{7 7}$
integrity theorem 151, 206
interior-point method 68
internal vertex
of directed walk 31
of walk 19
internally disjoint $s-t$ paths 132, 137-140, 142-147, 275-276
algorithm 137-138
complexity 139,276
min-max 132
planar 140 complexity 140
internally disjoint $T$-paths 1282
min-max 1282
internally disjoint $T$-paths theorem, Mader's ~ 1282
internally disjoint walks 20, 32
internally vertex-disjoint paths 132
internally vertex-disjoint $s-t$ paths 132, 137-140, 142-147, 275-276
algorithm 137-138
complexity 139, 276
min-max 132
planar 140
complexity 140
internally vertex-disjoint $T$-paths 1282 min-max 1282
internally vertex-disjoint walks 20, 32
intersect 17, 29
intersecting family 832-837
intersecting submodular function 832
intersecting supermodular function 837
intersection, contrapolymatroid $\sim$ 797-799, 818-819, 837
intersection, generalized polymatroid $\sim$ 847-849
intersection, matroid ~ 700-724, 739743, 768, 1026
complexity 700,707
weighted 707-712
intersection, optimization over polymatroid ~ 795-797
intersection, polymatroid ~ 795-819, 825-837, 840-841, 1020, 1024, 1026-1028
algorithm 805-819, 829-832, 835837, 840-841
intersection criterion 1310
intersection graph 1140, 1142
intersection theorem, matroid $\sim \mathbf{7 0 0}-$ 701, 704, 714-715, 768
interval colouring, $k$-~ 1151
interval graph 1140-1141
inverse Ackermann function 864
inverting component 469
irredundant system 64
isolated block 1077
isolated vertex 17
Jarník-Prim method 856-858, 872873, 875
job assignment 428-429
join 233, 510-515, 668, 960
maximum-size 511-515 min-max 511-515
join, 0-~ 1112-1113
join, 1-~ 1113
join, 2-~ 1112-1115
join, special 2-~ 1114
join, $T$-~ $\quad$ 485-519, 1417-1418
minimum-size 488-490, 502,504 min-max 489-490, 502, 504
shortest 485-486, 488-491, 501507, 517-518
algorithm 485-486 complexity 486,518 min-max 491
join-irreducible 233
join polytope, $T$-~ 490-492, 501-507, 517
adjacency 517
diameter 517
joins, disjoint $T$-~ $507-510,519,1413$, 1456
min-max $507-508$
jump system 722-723
$k$-arc-connected digraph 238, 1051 minimum-size 1051
$k$-arc-connected orientation 1044-1046 algorithm 1045 characterization 1044-1046
$k$-arc-connector 1058, 1060-1061 minimum-size 1060-1061 algorithm 1061 min-max 1060
$k$ arc-disjoint paths problem $\mathbf{1 2 2 3}$
$k$-chromatic graph 23, 1083
$k$-circuit 20
$k$-colourable graph 23, 1083
$k$-colouring 1083
$k$-commodity flow 1221-1222
$k$-commodity flow problem 1221
$k$-commodity flow problem, integer $\sim$ 1222
$k$-commodity flow problem, maximum-value ~ $\mathbf{1 2 2 2}$
$k$-commodity flow problem, undirected $\sim 1222$
$k$-commodity flow problem, undirected maximum-value $\sim 1222$
$k$-connected component 242
$k$-connected digraph 238, 1050-1051 minimum-size 1050-1051
$k$-connected digraph, strongly $\sim 238$, 1051
minimum-size 1051
$k$-connected graph 237, 1049-1050 minimum-size 1049-1050
$k$-connected orientation, strongly $\sim$ 1044-1046
algorithm 1045
characterization 1044-1046
$k$-connected subgraph 991 shortest 991
$k$-cover, directed cut ~ 950-951, 953954, 964-966, 968
minimum-size 950-951, 953-954
algorithm 953-954
min-max 950-951
minimum-weight 950, 953-954
algorithm 953-954
min-max 950
$k$-covering 36
$k$-cut 21, 33
$k$-cycle 1409
$k$-cycling matroid 1420
$k$ disjoint paths problem 1223
$k$-edge-chromatic graph 24
$k$-edge-colourable graph 24, 465
$k$-edge-colouring 321, 465
$k$-edge-connected component 248
$k$-edge-connected graph 238, 1050
minimum-size 1050
$k$-edge-connector 1062, 1065-1066
minimum-size 1065-1066 algorithm 1065 min-max 1065-1066
$k$-edge cover 578-579
in hypergraph 1429
minimum-size 579 min-max 579
$k$-edge cover, simple $\sim \mathbf{5 8 2}$
minimum-size 582 $\min -\max \quad 582$
$k$ edge-disjoint paths problem $\mathbf{1 2 2 3}$
$k$-factor $\mathbf{3 2 7}, \mathbf{3 4 0}, \mathbf{5 7 2}-574$
bipartite 327,340
characterization 327,340
characterization 572
$k$-flow, nowhere-zero $\sim 472$
$k$-flowing matroid 1420
$k$-flowing matroid, integer $\sim 1420$ 1421
$k$-graph 475, 644-645, 1454
$k$-heap 98-99, 128-129
$k$-interval colouring 1151
$k$-list-edge-colourable graph 335
$k$-matching $\quad 558-559$
in hypergraph 1378
maximum-size 558
$\min -\max 558$
$k$-matching, perfect $\sim$ 558-559
characterization 558
$k$-matching, perfect simple $\sim \equiv$ $k$-factor
$k$-matching, simple $\sim 572$
maximum-size 572 $\min -\max \quad 572$
$k$-matching, simple perfect $\sim \equiv$ $k$-factor
$k$-matching polytope 559
$k$-packing $\mathbf{3 6}$
$k$-perfect graph $\mathbf{1 1 5 0}$
$k$-regular edge function $\mathbf{2 6 9}$
$k$-regular graph 17
$k$-regularizable graph $\mathbf{3 3 0}, 561$
bipartite 330
characterization 330
characterization 561
$k$ shortest paths 129
$k$ shortest $s-t$ paths 105
$k$-stable set
in hypergraph 1429
$k$-sum of graphs $\mathbf{2 6}$
$k$-truncation of matroid 654
$k$-uniform hypergraph 36, 755
$k$-uniform matroid 654
$k$-valent vertex 17
$k$-vertex-colourable graph 23,1083
$k$-vertex-colouring 1083
$k$-vertex-connected digraph 238, 1050-1051
minimum-size 1050-1051
$k$-vertex-connected graph 237, 10491050
minimum-size 1049-1050
$k$-vertex-connector 1074-1075, 1077
minimum-size 1074-1075 $\min -\max \quad 1074-1075$
$k$-vertex cover
in hypergraph 1378
$k$-vertex-cut 22, 33
$k$ vertex-disjoint paths problem 1223
$K_{4}$-free graph 1120
$K_{4}$-subdivision, bad $\sim 1195$
$K_{4}$-subdivision, odd $\sim 1188,1201$, 1330, 1334
$K_{4}$-subdivision, totally odd $\sim 1196$
kernel 1126
kernel solvable graph 1126-1130
Klein bottle, graph on $\sim 1314-1316$, 1368
Kőnig property 536
Kőnig-Rado edge cover theorem 317320, 392, 703, 960, 972, 1023, 1135-1136, 1441
Kőnig's edge-colouring theorem 321322, 324-325, 331, 934, 1016, 1136, 1441
Kőnig's matching theorem $144,260-$ $263,275-277,281-284,304-305$,

392, 703, 783, 930, 1136, 1399, 1441
Kruskal's method 857-859, 871, 874
Kuratowski's theorem $\mathbf{2 6}$

Lagrangean multipliers 986
Lagrangean relaxation $\mathbf{9 8 5}-986$, 993
laminar collection of paths $\mathbf{2 7 0}$
laminar family $\mathbf{3 7}, \mathbf{2 1 4}-215,441$, $453,712,820,832$
laminar vector 616
last arc of walk 31
last edge of walk 19
last vertex of walk 19,31
lattice $\mathbf{8 1}, \mathbf{2 3 3}, \mathbf{6 6 8}-669,674-675$, 677, 681-682
lattice, distributive $\sim \mathbf{2 3 3}-235,1034$
lattice, dual $\sim \mathbf{8 1}$
lattice, geometric ~ 668
lattice, matroid ~ 668-669
lattice, modular ~ 674-675, 681
lattice, point ~ 668-669
lattice, upper semimodular $\sim \mathbf{6 6 9}$, 675, 677, 681-682
lattice family $\mathbf{8 2 6}-832,834-835$
lattice polyhedron $1025-1028$
leaf $\mathbf{4 3 4}$
leave $\mathbf{2 9}$
legal order $\mathbf{2 4 5}$
Lehman's theorem 1387-1392
length $\mathbf{1 3}$
of closed curve $\mathbf{1 3 5 6}$
of walk 19,31
length function $\mathbf{1 3}$
length of walk $\mathbf{9 6}$
length-width inequality $\mathbf{9 4}, \mathbf{2 2 1}, \mathbf{1 3 8 3}$
light 1127
Lin-Kernighan heuristic for the symmetric traveling salesman problem 996
line digraph $\mathbf{3 0}$
line graph 18
line of graph $\equiv$ edge of graph
line-perfect graph $\mathbf{1 1 4 5}$
linear extension 11
linear halfspace $\mathbf{5 9}$
linear matroid 654-655, 676-679, 728, 753
linear order 11
linear ordering problem $\mathbf{9 5 3}$
linear programming 61-63, 67-68, 84
linear programming, duality theorem of ~ 62-63
linear programming, integer $\sim 73-74-$ 84
linear programming duality equation 63
linear time, problem solvable in $\sim 47$
linear-time algorithm 47
linear-time solvable problem 47
linearly independent vectors $\mathbf{1 3}$
link 1337
linked list 48
linked list, doubly $\sim 48$-49
linked sets in digraph 140,659, 737
linking system 671
linklessly embeddable graph 956 -958
Lins' theorem 1299-1300
list 48-49
list, doubly linked $\sim$ 48-49
list, linked $\sim 48$
list-colouring $737-738,892$
of matroid 737-738
list-edge-colourable graph, $k$-~ 335
list-edge-colouring 335-336, 1455
bipartite 335-336
list-edge-colouring number 335, 482
literal 1084
local Euler condition 1366
lockable collection 1291-1292
longest $\equiv$ maximum-length 13
longest branching 895-896, 900-901
algorithm 895-896
min-max $900-901$
longest forest $860-861$
algorithm 860
min-max $860-861$
longest Hamiltonian circuit 996
longest path 114-117
acyclic 116-117
min-max $116-117$
NP-completeness 114-115
longest $R-S$ biforest $930-931$
algorithm 931
min-max 930
longest $R-S$ bifurcation 938-940
algorithm 940
min-max $\quad 938-940$
loop
in digraph 29
in graph 16
of matroid 651
loopless digraph 29
loopless graph 16
loopless vertex 16
lower ideal 11, 233, 1026, 1028
LP $\equiv$ linear programming
Lucchesi-Younger theorem 947-948, 972, 977, 1020, 1024, 1399-1400
$M$-alternating forest 420
$M$-alternating walk 416
$M$-augmenting path 259-260, 263264, 413
M-blossom 416
$M$-flower 416
$M$-posy $\mathbf{5 3 7}$
Mader matroid 1293-1294
Mader's disjoint $\mathcal{S}$-paths theorem 1280-1281
Mader's edge-disjoint $T$-paths theorem 1282-1283, 1289
Mader's internally disjoint $T$-paths theorem 1282
marginal component 1070
marriage theorem, defect form of Hall's ~ 380-381
marriage theorem, Hall's $\sim 379-380$, 392
matchable set $\mathbf{2 3}, \mathbf{2 6 2}, \mathbf{3 5 9}, \mathbf{4 5 0}-452$, 624
matchable set polytope $\mathbf{3 5 9}, \mathbf{4 5 0}-452$
bipartite 359
matched to $\mathbf{2 3}$
matching 23, 259-316, 321-347, 359-$362,378-409,413-460,536-539$, 1095, 1136, 1453
bipartite 260-316, 321-347, 359-
362, 378-409, 1136
history 278-284
maximum-size 260-267, 275-
278, 304-305, 316, 1136
algorithm 263-265, 277278, 316
complexity $267,276-277$ min-max 260-261
maximum-weight 285-288, 290300, 304-307
algorithm 286-288, 305-307
complexity 290
history 292-300
min-max 285-286
simplex method 290-291
exchange property $266-267$
in hypergraph 1377
in matroid 746
maximum-size 259-260, 315-316, 413-425, 429-437, 536-539, 1095, 1136
algorithm 415-421, 429-430, 436-437
complexity $422-423$
min-max $413-414$
maximum-weight 438-444, 448449, 453-460, 1453
algorithm 448-449, 456-458
complexity $458-459$
min-max $440-442$
nonbipartite 431-437
history 431-437
matching, 2-~ 341, 520-521, 523-526, 531-532
maximum-size $520-521,524-526$, 531-532
algorithm 521
min-max $520-521$
maximum-weight 523-524
algorithm 523
$\min -\max \quad 523$
matching, $b-\sim$ 337-347, 351-356, 358-
362, 546-576
bipartite 337-347, 353-356, 358362
maximum-size $338,342-343$, 358
algorithm 342-343
complexity 358
$\min -\max 338$
maximum-weight 337-338, 342343, 355-356
algorithm 342-343
complexity 355-356 min-max 338
maximum-size $351-352,546-547$, 556-557, 575-576
min-max 546-547, 557
maximum-weight 550-559, 561
algorithm $554-556,561$
complexity 559
min-max 550-553, 558
matching, basic path-~ $\mathbf{7 6 3}$
matching, bottleneck ~ 423
matching, capacitated $b-\sim$ 341-343, 357-358, 361, 562-568, 583
bipartite 341-343, 357-358
maximum-size 341-343, 358
algorithm 342-343
complexity 358
min-max 341-342 maximum-weight 342-343, 357
algorithm 342-343
complexity 357
min-max 342
maximum-size $562-564,567,583$ min-max 562-564
maximum-weight 566-567 algorithm 567 min-max 566
matching, capacitated perfect $b$-~ 342-343, 358, 564, 567
bipartite 342-343, 358 characterization 342 complexity 358
characterization 564
minimum-weight 567 algorithm 567
matching, $\mathcal{F}-\sim 545$
matching, fractional $\sim 521,1094$
in hypergraph 1378
matching, independent path-~ $\mathbf{7 6 4}$
matching, $k$-~ 558-559
in hypergraph 1378
maximum-size 558 min-max 558
matching, matroid ~ 746-765, 12831284
linear ~ 757-762 algorithm 757-762
NP-completeness 762-763
matching, maximum $\mathcal{F}$-~ 545
matching, path-~ 763-764
matching, perfect 2-~ 521, 524
characterization 521
complexity 521
minimum-weight 524
min-max 524
matching, perfect $\sim$ 23, 261-263, 267274, 276-279, 288-289, 304-307, 327, 414-415, 418, 422-423, 425-428, 430-436, 438-444, 448449, 453-460
algorithm 418
bipartite 261-263, 267-274, 276279, 288-289, 304-307
characterization 261
complexity 277
minimum-weight 288-289, 304307
algorithm 288, 305-307
min-max 288-289
regular 261-262, 267-274 algorithm 267-274
regular.history 278-279
characterization 414
complexity 422-423, 430
in hypergraph 1443
minimum-weight 438-444, 448449, 453-460
algorithm 448-449, 453-458
complexity 458-459
min-max 444
matching, perfect $b-\sim$ 338, 343, 358,
547, 553-554, 556-557, 567-568
bipartite 338, 343, 358
characterization 338
complexity 358
minimum-weight 343
algorithm 343
characterization 547,557
minimum-weight 553-554, 556
algorithm 556
min-max 553-554
matching, perfect $\mathcal{F}$-~ $\mathbf{5 4 5}$
matching, perfect $k$-~ 558-559
characterization 558
matching, perfect simple 2-~ $\equiv$ 2-factor
matching, perfect simple $b-\sim \equiv$ $b$-factor
matching, perfect simple $k$-~ $\equiv$ $k$-factor
matching, simple 2-~ 526-531, 535
maximum-size $526-528,535$
algorithm 528
min-max $\quad 526-527$
maximum-weight 531 min-max 531
matching, simple $b-\sim 339-343,354$, 358, 569-574, 582
bipartite 339-343, 354, 358
maximum-size $339,342-343$, 358
algorithm 342-343
complexity 358
min-max 339
maximum-weight 340-343
algorithm 342-343
min-max $340-341$
maximum-size $569,572-573,582$
algorithm 572
min-max 569
maximum-weight 571-573
algorithm 571-572 min-max 571
matching, simple $k-\sim 572$
maximum-size 572 $\min -\max \quad 572$
matching, simple perfect $2-\sim \equiv$ 2-factor
matching, simple perfect $b-\sim \equiv$ $b$-factor
matching, simple perfect $k-\sim \equiv$ $k$-factor
matching, stable $\sim$ 311-314
bipartite 311-314
algorithm 312-314
maximum-weight 313-314
algorithm 313-314
matching, triangle-free 2-~ 539-544
maximum-size $542-544$
matching, triangle-free perfect 2-~ 544
algorithm 544
matching-augmenting algorithm 418
matching-augmenting path 259
matching-covered graph $314, \mathbf{3 3 2}$,
426-428, 430, 512, 609-613, 617-619
matching forest 1005-1017
exchange property $1008-1011$
maximum-size $1006-1007,1016$ min-max 1006-1007
maximum-weight 1012-1016
$\min -\max \quad 1012-1016$
matching forest, perfect $\sim 1007-1008$
algorithm 1008
characterization 1007-1008
matching forest polytope 1011-1017 facets 1017
matching forests, covering by $\sim 1016$ $\min -\max 1016$
matching lattice $331-332$, 619-647
bipartite 331-332
matching lattice, perfect 2-~ $\mathbf{6 4 7}$
matching lattice, perfect $\sim$ 331-332, 619-647
bipartite 331-332
matching matroid 661, 1293-1294
matching number $\mathbf{2 3}, \mathbf{2 6 0}, 315-316$, 413-414
matching number, fractional $\sim 5 \mathbf{5 2 1}$, 1094
matching polytope $\mathbf{3 0 2}-305,310-311$, 439-448, 452, 459, 477-478
adjacency 444-445
bipartite $305,310-311$
diameter 445
facets 446-448
matching polytope, 2-~ $\mathbf{5 2 2}, 560$
facets 560
vertices 560
matching polytope, $b-\sim 338-339$, 547-553, 557, 559-561
adjacency 549
bipartite 338-339
diameter 549
facets 559
matching polytope, $c$-capacitated $b$-~ 342, 564-567
bipartite 342
facets 567
matching polytope, $c$-capacitated perfect $b-\sim \mathbf{5 6 5}$
matching polytope, fractional $\sim \mathbf{5 2 2}$
matching polytope, fractional $b-\sim 561$
vertices 561
matching polytope, $k$-~ 559
matching polytope, matroid $\sim 765$
matching polytope, perfect $2-\sim \mathbf{5 2 2}$ 524
matching polytope, perfect $\sim \mathbf{3 0 1}-$ 304, 307-310, 314, 327-328, 330-

331, 438-439, 443-445, 452, 459, 609-612
adjacency 307,445
bipartite 307-310, 314, 327-328, 330-331
diameter $307,445,452$
dimension 308, 609-612
matching polytope, perfect $b-\sim \mathbf{5 4 9}$, 553-554
matching polytope, simple 2-~ 528 531
facets 530
matching polytope, simple $b$-~ 340, 570-571, 574
adjacency 574
bipartite 340
facets 574
matching polytope, stable $\sim$ 312-313
bipartite 312-313
matching polytope, triangle-free 2-~ 539-544
facets 544
matching polytope theorem, Edmonds' ~ 440, 442-443
matching polytope theorem, Edmonds' perfect $\sim$ 438-439
matching problem, 3-dimensional $\sim$ 408
matching problem, matroid $\sim$ 745-765
matching space, perfect 2-~ 646-647
matching space, perfect $\sim$ 308-309, 331, 611-612
bipartite 308-309
dimension 308-309, 611-612
matching theorem, Kőnig's ~ 144, 260-263, 275-277, 281-284, 304305, 392, 703, 783, 930, 1136, 1399, 1441
matching vector, independent path-~ 764
matchings, covering by perfect $\sim 329-$ 331
bipartite 329-331 min-max 329-330
matchings, disjoint perfect $\sim 326-328$, 340
bipartite 326-328, 340 min-max 327
matchings, union of $\sim 340$
bipartite 340 min-max 340
matchoid problem $\mathbf{7 6 5}$
mate 23
matroid 651-765, 768, 775-776
history 672-687
matroid, algebraic ~ 656-657, 675679, 753-754, 765
matroid, binary ~ 655-656, 14061407, 1415, 1420-1427, 1456, 1461
matroid, cocycle ~ 657-658
matroid, cographic ~ 657-658
matroid, cycle $\sim \mathbf{6 5 7}$
matroid, cycle in binary $\sim \mathbf{6 5 5}$
matroid, dual ~652-653
matroid, Fano ~ 655
matroid, finite $\sim \mathbf{7 4 6}$
matroid, generalized $\sim \mathbf{8 5 2}$
matroid, graphic $\sim$ 657, 754-755, 823
matroid, induction of $\sim$ 736-737
matroid, infinite $\sim \mathbf{7 4 5}$
matroid, linear ~ 654-655, 676-679, 728, 753
matroid, matching ~ 661, 1293-1294
matroid, pseudomodular ~ 765
matroid, regular ~656, 1408, 1415, 1422
matroid, representable $\sim$ 654-655
matroid, strongly base orderable $\sim$ 738-743
matroid, transversal ~ 658-659, 727728, 739
matroid base covering theorem 727, 729
matroid base packing theorem $\mathbf{7 2 7}$
matroid cover 756-757
matroid intersection 700-724, 739-743, 768, 1026
complexity 700, 707
weighted 707-712
matroid intersection theorem 700-701, 704, 714-715, 768
matroid lattice 668-669
matroid matching 746-765, 1283-1284
linear ~ 757-762 algorithm 757-762
NP-completeness 762-763
matroid matching, fundamental circuit in $\sim \mathbf{7 4 7}$
matroid matching polytope 765
matroid matching problem 745-765
matroid matching theorem 751-752
matroid port 1407
matroid union 725-744
history 743-744
matroid union theorem $\mathbf{7 2 6}, 782$
matroids, union of $\sim \mathbf{7 2 6}$
max-biflow min-cut theorem 12551256
max-flow min-cut property 1383, 1385
max-flow min-cut property, $\mathbb{Q}_{+}$-~ 1383
max-flow min-cut property, $\mathbb{Z}_{+-} \sim$ 1397
max-flow min-cut theorem 150-151, 174, 198, 200, 205-206, 1020, 1399
max-potential min-work theorem 9697, 108, 972, 1026, 1413
maximal 10
maximal, inclusionwise $\sim 10$
maximal chain 235
maximal chains, disjoint $\sim 235$
min-max 235
maximal forest $\mathbf{8 5 5}$
maximum 10
maximum-capacity cut 486, 1345-1350
approximative algorithm 13451348
planar 486 algorithm 486
maximum-capacity path problem 117
maximum cut $\equiv$ maximum-size cut
maximum degree
of graph 17
of hypergraph 1380
maximum $\mathcal{F}$-matching $\mathbf{5 4 5}$
maximum flow 148-149-169, 173, 200-201, 1020, 1453
algorithm 151-160, 1453
complexity $160-161$
history 164-169
$\min -\max \quad 150-151$
planar 161-162
complexity 161-162
simplex method 162-163
maximum flow problem 149
maximum flow with upper and lower bounds 173
maximum reliability $117-118,866-867$
maximum reliability problem 117
maximum $s-t$ flow 149
maximum-size 2-matching 520-521, 524-526, 531-532
algorithm 521
min-max 520-521
maximum-size 2-stable set 531-532
algorithm 532
min-max 532
maximum-size antichain 218
min-max 218
maximum-size $b$-matching 338, 342343, 351-352, 358, 546-547, 556557, 575-576
bipartite 338, 342-343, 358
algorithm 342-343
complexity 358
min-max 338
min-max 546-547, 557
maximum-size capacitated $b$-matching 341-343, 358, 562-564, 567, 583
bipartite 341-343, 358 algorithm 342-343
complexity 358
min-max 341-342
min-max $\quad 562-564$
maximum-size chain 217
$\min -\max 217$
maximum-size clique 1084-1085, 11021185
in perfect graph 1106-1134, 1154 algorithm 1154
NP-completeness 1084-1085
maximum-size common independent set 700-701, 705-707, 710, 1026
algorithm 705-707
complexity 707, 710
min-max 700-701
maximum-size common partial transversal 394
min-max 394
maximum-size cut 1328,1350
complexity 1350
NP-completeness 1328
maximum-size directed cut 219-220
acyclic 219-220
min-max 219-220
maximum-size directed path 218
acyclic 218
min-max 218
maximum-size join 511-515
min-max 511-515
maximum-size $k$-matching 558 min-max 558
maximum-size matching 259-267, 275278, 304-305, 315-316, 413-425, 429-437, 536-539, 1095, 1136
algorithm 415-421, 429-430, 436437
bipartite 260-267, 275-278, 304305, 316, 1136
algorithm 263-265, 277-278, 316
complexity 267, 276-277
min-max 260-261
complexity $422-423$
min-max 413-414
maximum-size matching forest 1006 1007, 1016
min-max 1006-1007
maximum-size partial transversal 379381
min-max 379-381
maximum-size $R-S$ bifurcation $937-$ 938
min-max 938
maximum-size simple 2-matching 526528, 535
algorithm 528
min-max 526-527
maximum-size simple $b$-matching 339 , 342-343, 358, 569, 572-573, 582
algorithm 572
bipartite 339, 342-343, 358
algorithm 342-343
complexity 358
min-max 339
min-max 569
maximum-size simple $k$-matching 572 min-max 572
maximum-size stable set 315-317, 536-539, 972, 1023, 1084-1085,

1095, 1098-1185, 1196-1199, 1208-1212, 1217
bipartite $316-317,972,1023,1135$ algorithm 316 min-max 317
in claw-free graph 1208-1212 algorithm 1208-1212
in perfect graph 1106-1134, 11531154
algorithm 1153-1154
NP-completeness 1084-1085, 1217
maximum-size strong fractional stable set 1096
maximum-size triangle-free 2-matching 542-544
maximum-size union of forests 890
complexity 890
maximum-size $w$-stable set 318,534
bipartite 318 $\min -\max 318$
even $w \quad 534$ min-max 534
maximum-value fractional multiflow 1226-1227
maximum-value $k$-commodity flow problem 1222
maximum-value $k$-commodity flow problem, undirected ~ $\mathbf{1 2 2 2}$
maximum-value multiflow $1222,1225-$ 1228, 1230, 1237-1238, 12481249, 1255-1257, 1287-1288, 1290-1291, 1294-1295
maximum-value multiflow problem 1222
maximum-value multiflow problem, undirected ~ $\mathbf{1 2 2 2}$
maximum-weight 2-matching 523-524
algorithm 523
min-max 523
maximum-weight 2-stable set 578, 1091
algorithm 1091
min-max 578
maximum-weight antichain 220
$\min -m a x \quad 220$
maximum-weight $b$-matching 337-338, 342-343, 355-356, 550-559, 561
algorithm $\quad 554-556,561$
bipartite 337-338, 342-343, 355356
algorithm 342-343
complexity 355-356
min-max 338
complexity 559
min-max $550-553,558$
maximum-weight capacitated
b-matching 342-343, 357, 566567
algorithm 567
bipartite 342-343, 357
algorithm 342-343
complexity 357
min-max 342
min-max 566
maximum-weight clique 1097, 1157, 1159
in perfect graph 1157, 1159 algorithm 1157, 1159
maximum-weight common independent set 707-712, 714-715
algorithm 707-712
min-max 714-715
maximum-weight common independent set augmenting algorithm 707709
maximum-weight common partial transversal 397-399
algorithm 397
min-max 398-399
maximum-weight fractional stable set 1091
algorithm 1091
maximum-weight independent set 688692
algorithm 688-690
min-max 690-691
maximum-weight matching 285-288, 290-300, 304-307, 438-444, 448449, 453-460, 1453
algorithm 448-449, 456-458
bipartite 285-288, 290-300, 304307
algorithm 286-288, 305-307
complexity 290
history 292-300
min-max 285-286
simplex method 290-291
complexity 458-459
min-max 440-442
maximum-weight matching forest 1012-1016
min-max 1012-1016
maximum-weight partial transversal 382-383
algorithm 382
min-max 383
maximum-weight simple 2-matching 531
min-max 531
maximum-weight simple $b$-matching 340-343, 571-573
algorithm 571-572
bipartite 340-343
algorithm 342-343
min-max 340-341
min-max 571
maximum-weight stable matching 313-314
bipartite 313-314 algorithm 313-314
maximum-weight stable set 348,352 , 361, 1099-1101, 1155-1157, 1159, 1186-1195, 1213-1216
bipartite 348, 352, 361 algorithm 352, 361 min-max 348
in claw-free graph 1213-1216 algorithm 1213-1216
in perfect graph 1155-1157, 1159 algorithm 1155-1157, 1159
in t-perfect graph 1186-1195 algorithm 1186-1187
maximum-weight union of forests 890
complexity 890
maximum-weight $w$-stable set 348 , 578, 1200-1201
bipartite 348 min-max 348
even $w \quad 578$ min-max 578
mean capacity directed cut, minimum-~ 968
mean length 111, 500
mean length circuit, minimum-~ 500501
algorithm 500-501
mean length directed circuit, minimum-~ 111-112
algorithm 111-112
complexity 112
meet 17, 29, 233, 668
membership problem $\mathbf{7 0}$
Menger matroid 1293-1294
Menger's theorem 131-133, 142-147, 151, 164, 275-276, 720-721, 974, 1399, 1413
history 142-147
Menger's theorem, directed arc-disjoint version of $\sim \mathbf{1 3 2}$
Menger's theorem, directed internally vertex-disjoint version of $\sim \mathbf{1 3 2}$
Menger's theorem, directed vertex-disjoint version of $\sim$ 131-132
Mengerian hypergraph 1397-1402, 1460-1461
Mengerian hypergraph, binary $\sim$ 1409-1415
characterization 1409-1412
Mengerian matroid 1415
metric 10
metric, $\Gamma$-~ 1273, 1316
metric condition, $\Gamma$-~ 1273, 1316
metric cone 1345
metric inequalities 1345
Meyniel graph 1143-1145
MFMC $\equiv$ max-flow min-cut
min-flow max-cut theorem 220
minimal 10
minimal, inclusionwise $\sim 10$
minimal face of polyhedron $\mathbf{6 4}$
minimal system of inequalities $\mathbf{6 4}$
minimal totally dual integral 82
minimally crossing system of curves 1353
minimally imperfect graph 1107-1109, 1113, 1115-1125, 1145, 1150
minimally non-Mengerian hypergraph 1400
minimally nonideal hypergraph 1386, 1460
minimally nonpacking hypergraph 1401, 1461
minimization, submodular function $\sim$ 786-794
algorithm 786-792
complexity 791-792
minimization, symmetric submodular function ~ 792-793
algorithm 792-793
minimum 10
minimum-capacity cut 253-254
minimum-capacity $D_{0}$-cut 974
min-max 974
minimum-capacity directed cut 966967
source-sink connected 966-967 min-max 966-967
minimum-capacity odd cut 449
algorithm 449
minimum-capacity $r$-cut 907 min-max 907
minimum-capacity $s-t$ cut $150-156$, 159-162, 200-201, 974, 1020, 1413
algorithm 151-156, 159-160
complexity $160-161$
min-max 150-151
planar 161-162 complexity 161-162
minimum-capacity $T$-cut 498-500, 507-510
algorithm 499-500
minimum clique cover 1083
minimum colouring 23, 1083-1088, 1098, 1102-1185, 1206-1207
NP-completeness 1084-1085
of perfect graph 1106-1134, 11541155
algorithm 1154-1155
minimum-cost $b$-transportation 344 346, 356-357, 361-377
algorithm 344-346
complexity 356-357
minimum-cost $b$-transshipment 182183, 186-189, 191-192, 345-346
algorithm 182-183, 186-189
complexity 191
min-max 191-192
minimum-cost capacitated $b$-transportation 357-358, 361377
complexity 357-358
minimum-cost capacitated transportation 357-358, 361377
complexity 357-358
minimum-cost circulation 177-191, 195-197
algorithm 179-182, 189-190
complexity 190-191
simplex method 195
minimum-cost circulation problem 177
minimum-cost flow $177-191,195-197$
algorithm 185
simplex method 195
minimum-cost multiflow $1247-1248$, 1294-1295
minimum-cost $s-t$ flow problem 177
minimum-cost submodular flow 10191020, 1034
algorithm 1019-1020, 1034
$\min -\max 1019$
minimum-cost transportation 344-346, 356-357, 361-377
algorithm 344-346
complexity 356-357
min-max 345
minimum-cost transshipment 182-183, 186-189, 191-192, 345-346
algorithm 182-183, 186-189
complexity 191
min-max 191-192
minimum-cost union of $s-t$ paths 212-213
complexity 212-213
minimum cut $\equiv$ minimum-size cut 238
minimum degree of graph 17
minimum edge-colouring 24
minimum fractional colouring 1096, 1098
minimum fractional vertex-colouring 1096, 1098
minimum fractional weighted colouring 1097
NP-completeness 1097
minimum-mean capacity directed cut 968
minimum-mean length circuit 500-501 algorithm 500-501
minimum-mean length directed circuit 111-112
algorithm 111-112
complexity 112
minimum-requirement spanning tree 251-252
minimum-size 2-edge-connector 10621063
formula 1062-1063
minimum-size 2-edge cover 531-532
algorithm 532
$\min -\max 532$
minimum-size 2 -vertex-connector

$$
1077-1078
$$

min-max 1077-1078
minimum-size 2 -vertex cover $520-521$, 531-532
algorithm 521
min-max $520-521$
minimum-size b-edge cover 348 , 351352, 361, 575-576, 578
algorithm 576
bipartite $348,352,361$
algorithm 352, 361
min-max 348
min-max 576,578
minimum-size capacitated $b$-edge cover 350-351, 579-580, 583
algorithm 580
bipartite 350-351 $\min -\max \quad 350-351$
min-max $579-580$
minimum-size common spanning set 701
$\min -\max 701$
minimum-size cut 244-246
algorithm 244-246
minimum-size cut, all-pairs $\sim 248-251$
minimum-size cut problem, all-pairs $\sim$

## 248

minimum-size $D_{0}$-cut $973-976$
min-max $973-974$
minimum-size directed cut $962-968$
min-max $967-968$
source-sink connected 966 min-max 966
minimum-size directed cut cover 947949, 953-954, 956, 960, 972, 1020, 1024
algorithm 953-954
complexity 956
min-max $\quad 947-948$
minimum-size directed cut $k$-cover 950-951, 953-954
algorithm 953-954
min-max 950-951
minimum-size edge cover 315-317, 461-462, 464, 536-539, 972, 1023, 1095, 1135
algorithm 461-462
bipartite 316-317, 972, 1023, 1135 algorithm 316 $\min -\max 317$
$\min -m a x 461$
minimum-size feedback arc set 951953, 956-958
planar 958 min-max 958
minimum-size fractional vertex cover 1380-1381
in hypergraph 1380-1381
minimum-size $k$-arc-connected digraph 1051
minimum-size $k$-arc-connector $1060-$ 1061
algorithm 1061
$\min -\max 1060$
minimum-size $k$-connected digraph 1050-1051
minimum-size $k$-connected graph 1049-1050
minimum-size $k$-edge-connected graph 1050
minimum-size $k$-edge-connector $1065-$ 1066
algorithm 1065
min-max 1065-1066
minimum-size $k$-edge cover 579
min-max 579
minimum-size $k$-vertex-connected digraph 1050-1051
minimum-size $k$-vertex-connected graph 1049-1050
minimum-size $k$-vertex-connector 1074-1075
min-max 1074-1075
minimum-size $r$-cut $905-906,918,974$ algorithm 918
min-max 905-906
minimum-size $R-S$ bibranching 934935
algorithm 935
min-max 935
minimum-size $R-S$ biconnector 929
min-max 929
minimum-size $R-S$ bicut 940-943, 974
min-max 941-942
minimum-size $s-t$ cut 131-169
min-max 132
planar 139-140 complexity 139-140
minimum-size $S-T$ disconnecting vertex set 131-132
min-max 131-132
minimum-size $s-t$ vertex-cut 132
min-max 132
minimum-size simple 2-edge cover 535-536
algorithm 535
min-max 535
minimum-size simple $b$-edge cover 349-350, 581-582
algorithm 581-582
bipartite 349-350 min-max 349
min-max 581-582
minimum-size simple $k$-edge cover 582
min-max 582
minimum-size strong connector 972
min-max 972
minimum-size strongly $k$-connected digraph 1051
minimum-size $T$-cut 499, 507-508, 1413
min-max 499, 507-508
minimum-size $T$-join 488-490, 502, 504
min-max 489-490, 502, 504
minimum-size vertex cover 260-262, 265, 277, 304-305, 315-316, 536539, 1084-1085, 1095, 11031105, 1136, 1175, 1199-1200, 1380-1381
bipartite 260-262, 265, 277, 304305, 1136
algorithm 265
complexity 277
min-max 260-261
in hypergraph 1380-1381
NP-completeness 1084-1085
minimum-size vertex-cut 239-241
algorithm 239-241
complexity 241
minimum-size $w$-vertex cover 285-286, 289-290, 304, 523
bipartite 285-286, 289-290, 304
algorithm 289-290
min-max 285-286
even $w \quad 523$ $\min -\max 523$
minimum vertex-colouring 23, 10831088, 1098, 1102-1185, 12061207
NP-completeness 1084-1085
of perfect graph $1106-1134,1154-$ 1155
algorithm 1154-1155
minimum vertex-cut $\equiv$ minimum-size vertex-cut 237-238
minimum-weight 2-edge cover 534
min-max 534
minimum-weight 2-factor 528,531 , 986-987
algorithm 528
min-max 531
minimum-weight 2 -vertex cover 556557, 1094
algorithm 1094
min-max 557
minimum-weight $b$-edge cover 348 , 352-353, 577-578
algorithm 577-578
bipartite $348,352-353$ algorithm 352-353 $\min -\max 348$
min-max $577-578$
minimum-weight $b$-factor $341-343$, 571-572
algorithm 572
bipartite 341-343
algorithm 342-343
$\min -\max 341$
$\min -\max 571$
minimum-weight base 689-690, 692, 699
algorithm 689-690
$\min -\max 692$
minimum-weight capacitated $b$-edge cover 351-353, 580
algorithm 580
bipartite $351-353$
algorithm 351-353
$\min -\max 351$
min-max 580
minimum-weight capacitated perfect $b$-matching 567
algorithm 567
minimum-weight common base 710 , 715
algorithm 710
min-max 715
minimum-weight common spanning set 716
$\min -\max \quad 716$
minimum-weight common transversal 395-397
algorithm 396
min-max 396-397
minimum-weight directed cut cover 948-949, 953-954, 956, 972, 1020, 1024
algorithm 253-954
complexity 956
min-max $948-949$
minimum-weight directed cut $k$-cover 950, 953-954
algorithm 953-954
min-max 950
minimum-weight edge cover 317-318, 462-464
algorithm 317,462
bipartite 317-318
algorithm 317
min-max 318
min-max $\quad 462-464$
minimum-weight fractional vertex cover 1094
algorithm 1094
minimum-weight perfect 2-matching 524
min-max 524
minimum-weight perfect $b$-matching 343, 553-554, 556
algorithm 556
bipartite 343
algorithm 343
min-max 553-554
minimum-weight perfect matching 288-289, 304-307, 438-444, 448449, 453-460
algorithm 448-449, 453-458
bipartite 288-289, 304-307
algorithm 288, 305-307
min-max 288-289
complexity 458-459
min-max 444
minimum-weight simple 2-edge cover 535-536
algorithm 536
minimum-weight simple $b$-edge cover 350-353, 581
bipartite 350-353
algorithm 350-353
min-max 350
min-max 581
minimum-weight spanning set 693
$\min -m a x \quad 693$
minimum-weight transversal 382-383
algorithm 382
min-max 382-383
minimum-weight vertex cover 338, 343, 1159, 1187
bipartite 338, 343
algorithm 343
min-max 338
in perfect graph 1159 algorithm 1159
in t-perfect graph 1187
minimum-weight $w$-vertex cover 337338, 557-558
bipartite 337-338 min-max 338
even $w \quad 558$ min-max 558
minimum weighted colouring 10961097, 1157-1159
NP-completeness 1096-1097
of perfect graph 1157-1159 algorithm 1157-1159
minimum weighted vertex-colouring 1096-1097, 1157-1159
NP-completeness 1096-1097
of perfect graph 1157-1159 algorithm 1157-1159
minor
of graph 25, 1086
of hypergraph 1376
of matroid $\mathbf{6 5 4}$
of pair $G, T \quad 504$
of signed graph 1202, 1330
minor, $H \sim 25$
minor, odd ~ 1203, 1327, 1333,

## 1341

minor, proper $\sim 25$
misses vertex, edge $\sim \mathbf{1 7}$
misses vertex, matching $\sim 413$
mixed branching 926
mixed Chinese postman problem 518
mixed graph 30, 926, 1005-1017, 1037-1038, 1048, 1062, 1074
mixed graph, partitionable $\sim 1015$
mixed $r$-arborescence 926
modular function $\mathbf{7 6 6}$
modular lattice 674-675, $\mathbf{6 8 1}$
modular law 674
monotone ideal, down-~ 11
monotone ideal, up-~ 11
monotone in $\mathbb{R}_{+}^{n}$, down- $\sim \mathbf{6 6}$
monotone subset of $\mathbb{R}^{n}$, down-~ 65
monotone subset of $\mathbb{R}^{n}$, up-~ 65
monotone traveling salesman polytope

## 991

monotonic diameter
of polytope $\mathbf{9 9 0}$
most violated inequality problem 697698, $\mathbf{7 3 3}$
multicommodity flow $\equiv$ multiflow
multicut 254, 1230, 1295
complexity $254,1230,1295$
NP-completeness 254
multiflow 1221-1222-1325, 1334, 1341-1342, 1419-1427
history 1249-1250
in matroid 1419-1427
maximum-value 1222, 1225-1228, 1230, 1237-1238, 1248-1249, 1255-1257, 1287-1288, 12901291, 1294-1295
minimum-cost 1247-1248, 12941295
multiflow, directed ~ 1221, 1223,
1226-1228, 1234, 1241, 1243-
1244, 1248, 1262-1263, 1289,

1307, 1309-1310, 1322, 1325, 1368-1370
multiflow, feasible $\sim 1221$
multiflow, fractional ~1222, 12241231, 1234-1239, 1241, 12451249, 1270, 1272-1274, 1287, 1307-1308, 1317-1318, 1320, 1341-1342, 1354, 1357, 1361, 1368, 1459
algorithm 1225-1226
maximum-value $1226-1227$
multiflow, half-integer $\sim \mathbf{1 2 2 2}, 1230-$ 1231, 1234, 1236, 1238, 1251, 1253-1255, 1258, 1266, 12711274, 1288, 1290-1291, 1294, 1298, 1310, 1318, 1341-1342, 1361, 1459
complexity $1231,1234,1273,1310$
multiflow, integer ~ 1222-1225, 12301231, 1234-1235, 1239-1241, 1251, 1254-1255, 1257, 1266, 1271-1274, 1286-1288, 1290$1292,1307,1318,1320,1334$, 1342
complexity $1224-1225,1231,1251$
multiflow, quarter-integer $\sim 1231$, 1233-1234, 1236, 1274, 1318
multiflow problem 1221
multiflow problem, half-integer $\sim$ 1222
multiflow problem, integer $\sim \mathbf{1 2 2 2}$
multiflow problem, maximum-value $\sim$ 1222
multiflow problem, quarter-integer $\sim$ 1222
multiflow problem, undirected $\sim \mathbf{1 2 2 2}$
multiflow problem, undirected maximum-value $\sim \mathbf{1 2 2 2}$
multiflow problem in matroid 1419
multiflow subject to capacity 1221 1222
multiple arc 29
multiple edge of graph $\mathbf{1 6}$
multiplicity
of arc of digraph 29
of edge of graph $\mathbf{1 6}, 467$
of element of family $\mathbf{9}$

Nash-Williams covering forests theorem 934
Nash-Williams' covering forests theorem 879
Nash-Williams' disjoint trees theorem, Tutte-~ 877-878, 931, 1048
Nash-Williams' orientation theorem 1040-1044
near-bipartite graph 1217
near-perfect graph $\mathbf{1 1 2 0}$
nearest neighbour heuristic for the symmetric traveling salesman problem 995
nearest neighbour heuristic for the traveling salesman problem 999
negative edge of bidirected graph $\mathbf{5 9 4}$, 1201
neighbour 17, 22
neighbour, in~ 29
neighbour, out~ 29
net 1221-1222
network flow, polymatroidal ~ 10281029
network matrix 213-214
network synthesis 1049-1057
network synthesis problem 1051
node of digraph $\equiv$ vertex of digraph
node of graph $\equiv$ vertex of graph
non-Mengerian hypergraph, minimally $\sim 1400$
noncrossing condition 1320-1322
nondecreasing function $\mathbf{7 6 6}$
nonideal hypergraph, minimally $\sim$ 1386, 1460
nonincreasing function $\mathbf{7 6 6}$
nonpacking hypergraph, minimally $\sim$ 1401, 1461
nontrivial component of hypergraph 757
nontrivial cut $\mathbf{2 1}, \mathbf{3 3}, \mathbf{6 1 0}$
normal hypergraph 1432
north-west rule $\mathbf{3 7 2}$
nowhere-zero flow 470-473, 646, 1426-1427, 1454
in matroid 1426-1427
nowhere-zero flow in matroid 1454
nowhere-zero $k$-flow $\mathbf{4 7 2}$
NP 40-41, 71-72
NP, co-~ 42, 71-72

NP-complete problem 43-44, 72
objective function 63
odd antihole 1085, 1107
odd-blocking 516
odd circuit 1326-1329-1341, 1414
in bidirected graph 1201
in signed graph 1414
odd circuit cover 1327, 1329, 13351340, 1414
min-max 1335-1340
odd circuit cover polytope 1327
odd circuits, disjoint ~ 1335-1340
min-max 1335-1340
odd closed curve, doubly $\sim 1367$
odd component 20, 413
odd cut 449, 609
minimum-capacity 449 algorithm 449
odd cycle of binary hypergraph 1406 odd ear 425
odd ear-decomposition 425
odd edge set 1329
odd face of planar graph 1144
odd- $H \quad \mathbf{5 1 7}$
odd hole 1085, 1107
odd $K_{4}$-subdivision 1188, 1201, 1330, 1334
odd $K_{4}$-subdivision, totally $\sim 1196$
odd- $K_{n} \quad 1330$
odd minor 1203, 1327, 1333, 1341
odd path $\mathbf{5 1 5}-517, \mathbf{1 3 2 9}, 1456$
odd set $\mathbf{9}$
odd submodular function minimization 793-794, 842-845
algorithm 793-794, 842-845
odd walk 19, 593
oddly contractible to $K_{4} \quad 503$
Okamura-Seymour theorem 12961307
Okamura's theorem 1311-1318
openly disjoint paths $\equiv$ internally vertex-disjoint paths
optimization problem 69-71
optimum arborescence theorem 896, 898, 972, 1024, 1399
optimum bibranching theorem 937, 972, 1024
optimum ear-decomposition 512
optimum solution 14, 63
order, lexicographic ~ $\mathbf{1 1}$
order, linear ~ 11
order, partial ~ 11
order, pre-~ 11
order, pre-topological $\sim$ 89-90
algorithm 89-90
order, topological ~ 89-90
algorithm 90
order, total ~ $\mathbf{1 1}$
orderable graph, perfectly ~ $\mathbf{1 1 4 6}$
orderable matroid, strongly base $\sim$ 738-743
ordered set, partially ~11, 217-236, 1026-1028
orientation 29, 1035-1048, 1101-1102, 1204-1206
characterization 1035-1036, 1047
orientation, Eulerian ~ 34, 91 algorithm 91
orientation, $k$-arc-connected $\sim 1044-$ 1046
algorithm 1045
characterization 1044-1046
orientation, strongly connected $\sim$ 1037-1040, 1048
algorithm 1037-1038
characterization 1037-1040
orientation, strongly $k$-connected $\sim$ 1044-1046
algorithm 1045
characterization 1044-1046
orientation, well-balanced $\sim 1043$
orientation-preserving closed curve 1314
orientation-reversing closed curve 1299, 1314
orientation theorem, Nash-Williams' ~ 1040-1044
oriented matroid 1415-1416
orthogonally convex polyomino $\mathbf{1 1 4 9}$
outdegree of vertex 29
outer boundary of planar graph 26
outerplanar graph 28
outneighbour 29
output pairs 469
P 40
p-comparability graph 1149
$P$-critical graph 544
$P_{4}$-equivalent graphs $\mathbf{1 1 2 2}$
packing $\mathbf{3 6}$
packing, 2-~ 502
packing, $c-\sim \mathbf{3 6}$
packing, fractional $\sim \mathbf{3 6}$
packing, fractional $c-\sim 36$
packing, $k-\sim \mathbf{3 6}$
packing hypergraph 1401, 1460-1461
pairing 1040, 1302
pairing lemma 1302-1303
parallel arcs 29
parallel class of edges $\mathbf{1 6}$
parallel edges of graph 16
parallel elements of matroid 651
parallel extension $\mathbf{7 3 9}$
parallel forest-merging method $\mathbf{8 5 9}$, 871-874
parallel vertices of binary hypergraph 1409
parallelization of hypergraph 1376
paramodular collections $\mathbf{8 4 5}$
parent 99
parity 9
parity graph 1143,1145
parity graph, quasi-~ 1148
partial hypergraph 1439
partial order 11
partial $r$-arborescence $\mathbf{9 1 8}$
partial subhypergraph 1437, 1439
partial transversal 379-380-383
maximum-size $379-381$ min-max $379-381$
maximum-weight 382-383
algorithm 382
min-max 383
partial transversal, common $\sim 393$ 395, 397-399
maximum-size 394 $\min -\max 394$
maximum-weight 397-399 algorithm 397 min-max 398-399
partial transversal polytope $\mathbf{3 8 3}$-385
partial transversal polytope, common $\sim$ 399-400
partial transversals, covering by $\sim$ 386-387
$\min -\max 386$
partial transversals, covering by common ~ 402-403, 406
min-max 402
partially ordered set 11, 217-236, 1026-1028
partially ordered set, symmetric $\sim \mathbf{2 3 6}$
partition 10
partition, co~ 838, 841, 1047
partition, conjugate $\sim 230$
partition, proper $\sim \mathbf{8 3 4}$
partition, proper co~ $\mathbf{8 3 8}$
partition matroid 659
partition problem 46-47
NP-completeness 46-47
partitionable graph 1116-1118, 11231125,1166
partitionable mixed graph 1015
partitioning problem, set $\sim 1438$
path 19, 114-117
acyclic 116-117 longest 116-117 min-max 116-117
longest $114-115,117$
acyclic 117
NP-completeness 114-115
path, augmenting ~134, 151
path, bottleneck shortest $\sim 117-118$, 130
path, chordless $\sim \mathbf{1 9}$
path, directed ~31, 218
acyclic 218
maximum-size 218
$\min -\max 218$
path, $f$-augmenting $\sim 151$
path, Hamiltonian $\sim 24,34,114$
path, $M$-augmenting $\sim$ 259-260, 263264, 413
path, odd ~ 515-517, 1329, 1456
path, $r-\sim$ 1236, 1261, 1315
path, $\mathcal{S}-\sim 1280$
path, $s-t \sim 31,87-89,91-130,200-$
201, 487, 1026
shortest 87-89, 91-130, 200-201, 487, 1026
arbitrary-length 107-119, 487
acyclic 117
algorithm 109-111
complexity 112-113
planar 113

```
            complexity 113
            undirected 487
            algorithm 487
            complexity 487
    history 119-130
    nonnegative-length 96-106
                            algorithm 97-102
                            complexity 103-104
                            min-max 96-97
                            planar 104
                            complexity 104
    NP-completeness 114-115
    unit-length 87-89, 91-93, 95
        algorithm 88-89
        min-max 88
    zero-length 94
path, shortest ~ see shortest s-t
    path
path,T-~ 1279, 1289
path, weak T-~ }128
path in digraph 31
path-matching 763-764
path-matching, basic ~ 763
path-matching, independent ~ 764
path-matching vector, independent ~
        764
path polytope, s-t~ 198-203
    adjacency 202
    facets 202-203
    vertices 202
path problem, maximum-capacity ~
        117
paths, all-pairs shortest ~ 91-94, 104-
        105, 110-111, 113-114, 122, 125,
        127, 129, 517
    arbitrary-length 110-111, 113-114,
        517
        complexity 113
        planar 113-114
        undirected 517
            algorithm 517
            complexity }51
    nonnegative-length 104-105
        complexity 104-105
        planar 105
            complexity 105
    unit-length 91-93
        algorithm 91-92
        complexity 93
```

zero-length 94
complexity 94
paths, arc-disjoint ~132, 906, 1307
paths, arc-disjoint $s-t \sim 132,134-$ 140, 142-147, 151
algorithm 134-138
complexity 138-139
min-max 132
planar 139-140 complexity 139-140
paths, covering by $\sim 219,222-224$
algorithm 222-224
min-max 219
paths, covering by $s-t \sim$ 219-221
acyclic 219-220
min-max 219-220
min-max 220-221
paths, disjoint ~ 1223-1225, 1228, 1233-1234, 1239, 1242-1245, 1248, 1251, 1254, 1261-1265,
1267, 1271-1273, 1279-1296,
1298-1300, 1303-1304, 1307-
1311, 1313, 1315-1316, 1318,
1320-1325, 1352, 1361, 13661371, 1458-1459
complexity 1224-1225, 1243-1244, 1273, 1309, 1323, 1366, 1459
directed 1223-1225, 1243-1245, 1262-1263, 1289, 1309, 1322, 1368-1370
NP-completeness 1234
planar 1299
complexity 1299
paths, disjoint $\mathcal{S}$-~ 1280-1281
min-max 1280-1281
paths, disjoint $S-T \sim$ 131-132, 140147
exchange properties 140-141 min-max 131-132
paths, disjoint $T-\sim$ 1279-1295
algorithm 1283-1284
min-max 1279-1280
paths, edge-disjoint ~ 1253, 1255, 1285, 1296-1299, 1308, 13111313, 1318-1320
planar 1296-1299, 1308, 13111313, 1318-1320
algorithm 1298
characterization 1296-1298, 1308, 1311-1313, 13181320
complexity 1299
paths, edge-disjoint $s-t \sim 139,254$, 974, 1413
planar 139
complexity 139
paths, edge-disjoint $T$-~ 1282-1283, 1285-1286
algorithm 1285-1286
min-max $1282-1283$
paths, internally disjoint $s-t \sim 132$, 137-140, 142-147, 275-276
algorithm 137-138
complexity 139,276
$\min -\max 132$
planar 140
complexity 140
paths, internally disjoint $T$-~ 1282 $\min -\max 1282$
paths, internally vertex-disjoint $\sim \mathbf{1 3 2}$
paths, internally vertex-disjoint $s-t \sim$ $132,137-140,142-147,275-276$
algorithm 137-138
complexity 139,276
$\min -\max 132$
planar 140
complexity 140
paths, internally vertex-disjoint $T$-~ 1282
min-max 1282
paths, $k$ shortest $s-t \sim 105$
paths, openly disjoint $\sim \equiv$ internally vertex-disjoint paths
paths, union of $s-t \sim 210-213,227-$ 228
algorithm 212
complexity 212
min-max 210-211
minimum-cost $\quad 212-213$
complexity 212-213
paths, vertex-disjoint $\sim 1224-1225$, $1243,1299,1320-1323,1368-$ 1370
complexity $1224-1225,1243$
planar 1299, 1320-1323, 1368-1370
algorithm 1320-1323
characterization 1320-1323
complexity 1299
paths, vertex-disjoint $\mathcal{S}$-~ 1280-1281 $\min -\max$ 1280-1281
paths, vertex-disjoint $T$-~ 1279-1280, 1283-1284
algorithm 1283-1284
min-max 1279-1280
paths problem, arc-disjoint ~ $\mathbf{1 2 2 3}$
paths problem, disjoint $\sim \mathbf{1 2 2 3}$
fractional solution 1223
half-integer solution $\mathbf{1 2 2 3}$
paths problem, disjoint homotopic $\sim$ 1368
paths problem, edge-disjoint $\sim \mathbf{1 2 2 3}$
paths problem, homotopic edge-disjoint $\sim 1366$
paths problem, $k$ arc-disjoint $\sim \mathbf{1 2 2 3}$
paths problem, $k$ disjoint $\sim \mathbf{1 2 2 3}$
paths problem, $k$ edge-disjoint $\sim \mathbf{1 2 2 3}$
paths problem, $k$ vertex-disjoint $\sim$ 1223
paths problem, vertex-disjoint $\sim \mathbf{1 2 2 3}$
paths theorem, Gallai's disjoint $T$-~ 1279-1280
paths theorem, Mader's disjoint $\mathcal{S}$ - $\sim$ 1280-1281
paths theorem, Mader's edge-disjoint $T-\sim 1282-1283,1289$
paths theorem, Mader's internally disjoint $T$-~ $\mathbf{1 2 8 2}$
paths tree, shortest $\sim \mathbf{8 8}, 97-101$, $105,107,109,118,871$
paw 1121
paw-free graph 1121
pendant block 1077
perfect 2-matching $\mathbf{5 2 1}, 524$
characterization 521
complexity 521
minimum-weight 524
min-max 524
perfect 2-matching, simple $\sim \equiv$ 2-factor
perfect 2-matching, triangle-free $\sim 544$ algorithm 544
perfect 2-matching lattice $\mathbf{6 4 7}$
perfect 2-matching polytope $\mathbf{5 2 2 - 5 2 4}$
perfect 2-matching space 646-647
perfect $b$-matching $\mathbf{3 3 8}, 343,358,547$, 553-554, 556-557, 567-568
bipartite 338, 343, 358
characterization 338
complexity 358
minimum-weight 343 algorithm 343
characterization 547, 557
minimum-weight 553-554, 556
algorithm 556
min-max 553-554
perfect $b$-matching, capacitated $\sim$ 342-343, 358, 564, 567
bipartite 342-343, 358 characterization 342 complexity 358
characterization 564
minimum-weight 567 algorithm 567
perfect $b$-matching, simple $\sim \equiv$ $b$-factor
perfect $b$-matching polytope $\mathbf{5 4 9}, 553-$ 554
perfect $b$-matching polytope, $c$-capacitated $\sim \mathbf{5 6 5}$
perfect claw-free graph 1120
perfect $\mathcal{F}$-matching $\mathbf{5 4 5}$
perfect graph 1106-1107-1185, 1458
history 1176-1185
perfect graph, h-~ $\mathbf{1 2 0 7}$
perfect graph, $k$-~ $\mathbf{1 1 5 0}$
perfect graph, line-~ 1145
perfect graph, near-~ 1120
perfect graph, strongly $\sim$ 1144-11451146
perfect graph, strongly t-~ 11871195, 1458
perfect graph, super~ $\mathbf{1 1 5 1}$
perfect graph, t-~ 1099, 1186-1195, 1207, 1349-1350, 1458
perfect graph, trivially $\sim \mathbf{1 1 4 1}$
perfect graph conjecture, strong $\sim$ 1107, 1123-1124, 1178-1181, 1184-1185
perfect graph conjecture, weak ~ $\mathbf{1 1 0 7}$
perfect graph theorem 1108-11091110, 1125-1126, 1182-1185
perfect graph theorem, strong $\sim 1085$, 1107, 1116, 1120-1127, 1145
perfect hypergraph 1431-1434
perfect $k$-matching 558-559
characterization 558
perfect $k$-matching, simple $\sim \equiv$ $k$-factor
perfect matching 23, 261-263, 267274, 276-279, 288-289, 304-307, 327, 414-415, 418, 422-423, 425-428, 430-436, 438-444, 448449, 453-460
algorithm 418
bipartite 261-263, 267-274, 276279, 288-289, 304-307
characterization 261
complexity 277 minimum-weight 288-289, 304307
algorithm 288, 305-307
min-max 288-289
regular 261-262, 267-274
algorithm 267-274
regular.history 278-279
characterization 414
complexity 422-423, 430
in hypergraph 1443
minimum-weight 438-444, 448449, 453-460
algorithm 448-449, 453-458
complexity 458-459
min-max 444
perfect matching cone 644
perfect matching forest $1007-1008$
algorithm 1008
characterization 1007-1008
perfect matching lattice 331-332, 619-647
bipartite 331-332
perfect matching on set of vertices $\mathbf{6 7 0}$
perfect matching polytope 301-304, 307-310, 314, 327-328, 330-331, 438-439, 443-445, 452, 459, 609-612
adjacency 307, 445
bipartite 307-310, 314, 327-328, 330-331
diameter $307,445,452$
dimension 308, 609-612
perfect matching polytope theorem, Edmonds' ~ 438-439
perfect matching space 308-309, 331, 611-612
bipartite 308-309
dimension 308-309, 611-612
perfect matchings, covering by $\sim 329$ 331
bipartite 329-331
min-max 329-330
perfect matchings, disjoint $\sim 326-328$, 340
bipartite 326-328, 340 min-max 327
perfect matrix $\mathbf{1 4 3 7}$
perfect simple 2-matching $\equiv 2$-factor
perfect simple $b$-matching $\equiv b$-factor
perfect simple $k$-matching $\equiv k$-factor
perfectly orderable graph 1146
permutation graph 1138
permutation matrix $\mathbf{1 3}, 302$
Petersen graph 26-27, 466-467, 474, 477-478, 483-484, 497, 509, 620-621-622, 634, 636, 644, 984, 987, 992, 1404, 1408-1409, 1426, 1461
Petersen's theorem 415
planar digraph $\mathbf{3 5}, 951,1323,1325$
planar graph 25-26, 104-105, 113-114, 139-140, 145, 161-162, 164, 251, 470-471, 476, 480-484, 486, 494497, 518, 658, 951-952, 959, 963, 1084-1085, 1087, 1097, 1121, 1188, 1224-1225, 1234-1236, 1239, 1242-1244, 1247, 1257, 1265, 1296-1325, 1328-1329, 1341, 1345, 1361-1371, 1459
planar graph, straight-line $\sim 1367$
pluperfect graph, $\gamma-\sim \mathbf{1 1 8 2}$
point lattice 668-669
point of digraph $\equiv$ vertex of digraph
point of graph $\equiv$ vertex of graph
point of lattice 668,677
pointed polyhedron 64
pointer 48-49
polar 65
polar cone 65
polarity 65
polyhedral combinatorics $\mathbf{2 , 6 - 7}$
history 6-7
polyhedral cone $\mathbf{6 0}$
polyhedron 60-61, 84
polyhedron, integer $\sim \mathbf{7 4}-81$
polyhedron, pointed $\sim \mathbf{6 4}$
polyhedron, rational $\sim 61$
polyhedron determined by $\mathbf{6 0}$
polymatroid 766-767-852
adjacency 777
facets 777
operations on 781-782
vertices 776-777
polymatroid, characterization of $\sim$ 779-780
polymatroid, contra~ 774-775, 798799, 818-819, 837
intersection with polymatroid 798799, 818-819, 837
polymatroid, dimension of generalized $\sim 849$
polymatroid, extended $\sim \mathbf{7 6 7}$
polymatroid, face of $\sim 778$
polymatroid, generalized $\sim$ 845-849, 1020-1021
polymatroid, induction of $\sim 782-783$
polymatroid, optimization over $\sim 771-$ 773
polymatroid, structure of $\sim 776-778$
polymatroid intersection 795-819, 825-837, 840-841, 1020, 1024, 1026-1028
algorithm $\quad 805-819,829-832,835-$ 837, 840-841
polymatroid intersection, contra~ 797-799, 818-819, 837
polymatroid intersection, generalized $\sim$ 847-849
polymatroid intersection, optimization over ~ 795-797
polymatroid intersection theorem $\mathbf{7 9 6}$
polymatroidal network flow 1028-1029
polynomial time, problem solvable in $\sim$ 39-40
polynomial time, problem solvable in strongly $\sim 47$
polynomial-time algorithm $\quad 39-40$
polynomial-time algorithm, semi-strongly $\sim 48$
polynomial-time algorithm, strongly $\sim$ 47-48, 69-70
polynomial-time algorithm, weakly $\sim$ 48
polynomial-time solvable problem $\mathbf{3 9}$ 40
polynomial-time solvable problem, strongly $\sim 47$
polynomial-time solvable system of polyhedra 69
polyomino 1149
polytope 60-61, 84
polytope, $0,1 \sim \mathbf{7 5}$
positive edge of bidirected graph $\mathbf{5 9 4}$, 1201
postman problem, asymmetric $\sim \mathbf{5 1 8}$
postman problem, Chinese ~ 487488, 518-519
algorithm 487-488
complexity 488,518
history 519
windy postman problem 518
postman problem, directed Chinese ~ 192, 518
postman problem, windy $\sim 518$
posy, $M$-~ $\mathbf{5 3 7}$
potential 107-110, 126, 287
pre-order 11
pre-topological order 89-90
algorithm 89-90
preflow 156
prescribed degrees, subgraph with $\sim$ 586
Prim's method 856-858, 872-873, 875
primal-dual iteration $\mathbf{7 3}$
primal-dual method 72-73, 305-307
primal integrality $\mathbf{7 7}$
primal problem 63
prism 517
problem 40
problem, decision $\sim 40$
problem, dual $\sim 63$
problem, input of $\sim \mathbf{4 0}$
problem, instance of $\sim \mathbf{4 0}$
problem, linear-time solvable $\sim \mathbf{4 7}$
problem, polynomial-time solvable $\sim$ 39-40
problem, primal ~ 63
problem, strongly polynomial-time solvable $\sim 47$
problem, well-characterized $\sim 42$
problem solvable in linear time 47
problem solvable in polynomial time 39-40
problem solvable in strongly polynomial time 47
processor, two-~ 428
processor scheduling, two-~ 428-429
product, tensor ~
of matrices 12, 1168
of vectors 12, 1161
product of graphs, strong ~ 1167
profit 13
of circuit 1199
of edge 1199
of family of edges and circuits 1199
profit function 13
projection 417, 609, 622
projective plane, graph on $\sim 1299-$ 1301
proper copartition $\mathbf{8 3 8}$
proper ear $\mathbf{2 5 2}$
proper ear-decomposition 252
proper minor $\mathbf{2 5}$
proper partition $\mathbf{8 3 4}$
proper subgraph 18,30
proper subset $\mathbf{9}$
proper substructure $\mathbf{9}$
pseudomodular matroid $\mathbf{7 6 5}$
push 157
push, saturating ~ 158
push-relabel method 156-157-159
$\mathbb{Q}_{+}$-max-flow min-cut property 1383
quarter-integer multiflow 1231, 12331234, 1236, 1274, 1318
quarter-integer multiflow problem 1222
quarter-integral, totally dual $\sim \mathbf{8 1}$
quasi-balanced hypergraph 1383
quasi-parity graph 1148
quotient space 13
$r$-arborescence 34, 254, 893-897, 902-903, 972, 1024, 1399
shortest 893-897, 902-903, 972,
1024
algorithm 893-895 complexity 902 min-max 896
$r$-arborescence, mixed $\sim \mathbf{9 2 6}$
$r$-arborescence, partial $\sim 918$
$r$-arborescence polytope $\mathbf{8 9 7}-899,907$
$r$-arborescences, capacitated disjoint $\sim$ 922
complexity 922
$r$-arborescences, covering by $\sim 911-$ 913
min-max 912-913
$r$-arborescences, disjoint $\sim$ 905-907, 918-922, 925, 974, 1078-1079
algorithm 918-921
complexity 921-922
min-max 905-907
$r$-arborescences, union of $\sim$ 913-915
min-max 913, 915
$r$-coarborescence 941
$r$-cut 896, 905-907, 918, 974, 1399
minimum-capacity 907 min-max 907
minimum-size $905-906,918,974$
algorithm 918
min-max 905-906
$r$-cut polytope $\mathbf{9 0 7}$
$r$-cuts, disjoint $\sim$ 896-897, 972, 1024
$\min -m a x \quad 896$
$r$-edge-connected graph $\mathbf{1 0 5 5}, 1067$
$r$-edge-connector 1067
$r$-path 1236, 1261, 1315
$R-S$ bibranching 934-945, 972, 1024
minimum-size 934-935
algorithm 935
min-max 935
shortest 935-937, 972, 1024
algorithm 937
min-max 936-937
$R-S$ bibranching polytope 937,942
$R-S$ bibranchings, disjoint $\sim 940-$ 944, 974
min-max 941-942
$R-S$-bibranchings, disjoint $\sim 942$
algorithm 942
$R-S$ biconnector 928-930, 944
minimum-size 929 min-max 929
shortest 928-930
algorithm 930 min-max 929-930
$R-S$ biconnector polytope 929-930
$R-S$ biconnectors, disjoint $\sim 931-$ 934, 944
algorithm 933
min-max 933
$R-S$ bicut 935, 940-943, 972, 974, 1024
minimum-size 940-943, 974 min-max 941-942
$R-S$ bicuts, disjoint $\sim$ 937, 972, 1024
min-max 937
$R-S$ biforest 930-931, 944-945
longest 930-931
algorithm 931
min-max 930
$R-S$ biforest polytope 931
$R-S$ biforests, covering by $\sim 934$, 944-945
algorithm 934
min-max 934
$R-S$ bifurcation 937-940, 944-945, 1016
longest 938-940
algorithm 940
min-max 938-940
maximum-size 937-938 min-max 938
$R-S$ bifurcation polytope 940, 944
$R-S$ bifurcations, covering by $\sim 943-$ 945
algorithm 944
min-max 943-944
ractional stable set, strong $f \sim 1096$, 1098-1099
maximum-size 1096
Rado-Hall theorem $\mathbf{7 0 2}$
Rado's theorem $\mathbf{7 0 2}$
RAM $\equiv$ random access machine
random access machine 39
rank
of element of partially ordered set 668
of matroid 651
rank facet 1216
rank function
of matroid 651, 664-665
of polymatroid $\mathbf{7 7 9}$
rational affine halfspace $\mathbf{6 0 7}$
rational polyhedron 61
ray of polyhedron, extremal $\sim \mathbf{6 5}$
reach 31
reachable from $\mathbf{3 1}$
reachable to $\mathbf{3 1}$
realizable as the distance function of a planar graph with boundary $C$ 1306
realizable function 1306
realizable function, exactly $\sim 1051$
realization 1051
realization, exact $\sim 1051$
realization problem 1055
NP-completeness 1055
rectangular grid $\mathbf{1 3 2 3}$
rectilinearly visible corners 1324
Rédei's theorem 232, 1101
reduced $T$-border $\mathbf{5 0 7}$
reducible to problem, problem $\sim 43$
region, feasible $\sim 63$
regular bipartite perfect matching 261-262, 267-274, 278-279
algorithm 267-274
history 278-279
regular edge function, $k-\sim 269$
regular edge set, almost $\sim \mathbf{2 6 8}$
regular graph $\mathbf{1 7}$
regular graph, $k$-~ $\mathbf{1 7}$
regular matroid 656, 1408, 1415, 1422
regular system of closed curves $\mathbf{1 3 5 3}$
regularizable graph 560-561
characterization 560-561
regularizable graph, $k$-~ 330, 561
bipartite 330
characterization 330
characterization 561
Reidemeister move 1353-1354
relabel 157
relaxation 984, 992, 1347
reliability $\mathbf{8 6 6}$
of path 117, $\mathbf{8 6 6}$
of vertex pair $\mathbf{8 6 6}$
reliability, maximum $\sim$ 117-118, 866867
reliability problem, maximum $\sim \mathbf{1 1 7}$
replicating vertex
of graph 1109
replicating vertex by factor 1109
replication lemma 1110-1111
representable matroid 654-655
representation of matroid $\mathbf{6 5 4}$
representatives, common system of restricted $\sim 407$
characterization 407
representatives, system of distinct $\sim \equiv$ transversal
representatives, system of restricted $\sim$ 388, 407
characterization 388
represented by vectors, linear matroid $\sim 654$
residual graph $\mathbf{1 5 0}$
resigning of signed graph 1202
restricted linear program $\mathbf{7 3}$
restricted representatives, common system of $\sim 407$
characterization 407
restricted representatives, system of $\sim$ 388, 407
characterization 388
restriction
of hypergraph 1376
of matroid 653
reverse digraph $\mathbf{3 0}$
reverse walk 19
rigid-circuit graph $\equiv$ chordal graph
rigid graph $\mathbf{8 2 4}$
rigidity 824
Robbins' theorem 1037-1038
root
of branching 893
of matching forest 1005
of rooted forest 34
of rooted tree $\mathbf{3 4}$
root, co~ 942
root vector $\mathbf{9 2 3}$
rooted at, arborescence ~ 893
rooted at, rooted tree $\sim \mathbf{3 4}$
rooted forest $\mathbf{3 4}$
rooted tree $\mathbf{3 4}$
rooted tree-representation 215
Rothschild-Whinston theorem 12521254
routing problem, channel ~ 1323
row strategy 296
run 19, 29, 31
$S$-alternating walk 1208
$S$-augmenting path 1208
$\mathcal{S}$-path 1280
$\mathcal{S}$-paths, disjoint $\sim 1280-1281$
min-max 1280-1281
$\mathcal{S}$-paths, vertex-disjoint $\sim 1280-1281$ min-max 1280-1281
$\mathcal{S}$-paths theorem, Mader's disjoint $\sim$ 1280-1281
$s-t$ connector 203
$s-t$ connector polytope 203-204 dimension 203
$S-T$ cut 21, 33
$s-t$ cut 21, 33, 87, 131-132-169, 200-201, 974, 1020, 1413 minimum-capacity $150-156,159-$ $161,200-201,974,1020,1413$
algorithm 151-156, 159-160
complexity 160-161
min-max $150-151$
minimum-size 131-169
$\min -\max 132$
planar 139-140, 161-162
minimum-capacity $161-162$
complexity $161-162$
minimum-size $139-140$ complexity 139-140
$s-t$ cut polytope 199,203
adjacency 203
vertices 203
$s-t$ cuts, disjoint $\sim$ 87-88, 96-97, 126, 1026, 1413
min-max $88,96-97$
$s-t$ cuts, union of disjoint $\sim 211-212$
algorithm 212
min-max 211-212
$S-T$ disconnecting arc set $\mathbf{3 3}$
$s-t$ disconnecting arc set $\mathbf{3 3}$
$S-T$ disconnecting edge set $\mathbf{2 1}$
$s-t$ disconnecting edge set 21
$S-T$ disconnecting vertex set 131-132
minimum-size $131-132$
min-max 131-132
22, 34,
disconnecting vertex set $\equiv s-t$
vertex-cut 22, 33
$s-t$ flow, maximum $\sim 149$
$s-t$ flow problem, minimum-cost $\sim$ 177
$S-T$ path 31
$s-t$ path 31, 87-89, 91-130, 200-201, 487, 1026
shortest $87-89,91-130,200-201$, 487, 1026
arbitrary-length $107-119,487$
acyclic 117
algorithm 109-111
complexity 112-113
planar 113
complexity 113
undirected 487
algorithm 487
complexity 487
history 119-130
nonnegative-length 96-106
algorithm 97-102
complexity 103-104
min-max $\quad 96-97$
planar 104
complexity 104
NP-completeness 114-115
unit-length 87-89, 91-93, 95
algorithm $\quad 88-89$
$\min -\max 88$
zero-length 94
$s-t$ path polytope $198-203$
adjacency 202
facets 202-203
vertices 202
$s-t$ paths, arc-disjoint $\sim 132,134-$
140, 142-147, 151
algorithm 134-138
complexity 138-139
min-max 132
planar 139-140
complexity 139-140
$s-t$ paths, covering by $\sim 219-221$
acyclic 219-220
$\min -\max \quad 219-220$
min-max 220-221
$S-T$ paths, disjoint $\sim 131-132,140-$ 147
exchange properties $140-141$
min-max 131-132
$s-t$ paths, edge-disjoint $\sim 139,254$, 974, 1413
planar 139
complexity 139
$s-t$ paths, internally disjoint $\sim 132$, 137-140, 142-147, 275-276
algorithm 137-138
complexity 139, 276
min-max 132
planar 140
complexity 140
$s-t$ paths, internally vertex-disjoint $\sim$ 132, 137-140, 142-147, 275-276
algorithm 137-138
complexity 139,276
min-max 132
planar 140 complexity 140
$s-t$ paths, $k$ shortest $\sim 105$
$s-t$ paths, union of $\sim 210-213,227-$ 228
algorithm 212
complexity 212
min-max $\quad 210-211$
minimum-cost $212-213$
complexity 212-213
$S-T$ separating edge set $\mathbf{2 1}$
$s-t$ separating edge set 21
$S-T$ separating vertex set $\mathbf{2 2}, \mathbf{3 4}$
$s-t$ separating vertex set $\mathbf{2 2}, \mathbf{3 3}$
$S-T$ vertex-cut 22, $\mathbf{3 4}$
$s-t$ vertex-cut 22, 33, 132
minimum-size 132
min-max 132
$S-T$ walk 19, 31
$S-t$ walk 19
$s-T$ walk 19
$s-t$ walk 19, 31
sandwich theorem, Frank's discrete $\sim$ 799
satisfiability problem 44-46
NP-completeness 44-45
satisfiability problem, $3-\sim 46$
NP-completeness 46
satisfiable word 46
saturating push 158
scaling, capacity-~159-160
scanning vertex $\mathbf{8 9}$
Scarf's lemma 1128-1129
scheduling, two-processor $\sim 428-429$
SDR $\equiv$ transversal
search, breadth-first $\sim \mathbf{8 8}$
search, depth-first $\sim \mathbf{8 9}$
semi-strongly polynomial-time algorithm 48
semidefinite programming 991, 11521176, 1345-1348
semimodular lattice, upper $\sim \mathbf{6 6 9}$, 675, 677, 681-682
seminormal hypergraph 1402
sending flow over path 185
separates pair, curve $\sim 1321$
separates pair, set $\sim \mathbf{9}$
separates set, set $\sim \mathbf{9}$
separates sets, vertex set $\sim \mathbf{2 2}, \mathbf{3 4}$
separates vertex pair, edge set $\sim \mathbf{2 1}$
separates vertex sets, edge set $\sim \mathbf{2 1}$
separates vertices, vertex set $\sim \mathbf{2 2}, \mathbf{3 3}$
separating edge set, $S-T \sim \mathbf{2 1}$
separating edge set, $s-t \sim \mathbf{2 1}$
separating vertex set, $S-T \sim \mathbf{2 2}, \mathbf{3 4}$
separating vertex set, $s-t \sim \mathbf{2 2}, \mathbf{3 3}$
separation 22
separation problem 69-71
serial vertex in hypergraph 1434
serialization of hypergraph 1434
series elements of matroid $\mathbf{6 5 3}$
series-parallel graph $\mathbf{2 8}$
set covering problem 1438
set function $\mathbf{7 6 6}$
set packing problem 1104, 1382
set partitioning problem 1438
Seymour graph 518
Shannon capacity 1167-1171, 11761178, 1184-1185
shore 610
shortest $\equiv$ minimum-length 13
shortest 1-tree 985
shortest arborescence 902
min-max 902
shortest circuit of matroid 672
NP-completeness 672
shortest directed 1-tree 993
shortest directed circuit 94
shortest feedback arc set 951-953 complexity 951
shortest Hamiltonian circuit 981-982
shortest $k$-connected subgraph 991
shortest path see shortest $s-t$ path
shortest path, bottleneck ~117-118, 130
shortest paths, all-pairs $\sim 91-94,104-$ $105,110-111,113-114,122,125$, 127, 129, 517
arbitrary-length 110-111, 113-114, 517
complexity 113
planar 113-114
undirected 517
algorithm 517
complexity 517
nonnegative-length 104-105
complexity 104-105
planar 105
complexity 105
unit-length 91-93
algorithm 91-92
complexity 93
zero-length 94
complexity 94
shortest paths tree $\mathbf{8 8}, \mathbf{9 7}-101,105$, 107, 109, 118, 871
shortest $r$-arborescence 893-897, 902903, 972, 1024
algorithm 893-895
complexity 902
min-max 896
shortest $R-S$ bibranching 935-937, 972, 1024
algorithm 937
min-max 936-937
shortest $R-S$ biconnector 928-930
algorithm 930
min-max 929-930
shortest $s-t$ path 87-89, 91-130, 200-201, 487, 1026, 1413
arbitrary-length 107-119, 487 acyclic 117
algorithm 109-111
complexity 112-113
planar 113
complexity 113
undirected 487
algorithm 487
complexity 487
history 119-130
nonnegative-length 96-106
algorithm 97-102
complexity 103-104
min-max 96-97
planar 104
complexity 104
NP-completeness 114-115
unit-length 87-89, 91-93, 95
algorithm 88-89
min-max 88
zero-length 94
shortest $s-t$ paths, $k \sim 105$
shortest spanning tree 855-860, 862-

$$
866,868-869,871-876
$$

algorithm 856-860
complexity 864-865
history 871-876
min-max $862-863$
uniqueness 868-869
shortest strong connector 969-973, 1024
algorithm 971-972
min-max 971-972
shortest $T$-join 485-486, 488-491, 501507, 517-518, 1413, 1417-1418
algorithm 485-486
complexity 486,518
min-max 491
shortest tree see shortest spanning tree
shrinking 416
sift-down 99
sift-up 99
signature method 291
signed graph 1329
signed graph, equivalent $\sim 1329$
signed graph, resigning of $\sim \mathbf{1 2 0 2}$
signed graph of bidirected graph, underlying ~ 1201
signing 1329
signing, equivalent $\sim 1329$
similar vertices 1209
similarity class 1209
simple 2-edge cover 535-536
minimum-size 535-536
algorithm 535
min-max 535
minimum-weight 535-536
algorithm 536
simple 2-edge cover polytope $\mathbf{5 3 6}$
simple 2-matching 526-531, 535
maximum-size $526-528,535$
algorithm 528
min-max 526-527
maximum-weight 531
min-max 531
simple 2-matching, perfect $\sim \equiv$ 2-factor
simple 2-matching polytope $\mathbf{5 2 8}$-531 facets 530
simple b-edge cover 349-354, 581-582 bipartite 349-354
minimum-size 349-350 min-max 349
minimum-weight 350-353
algorithm 350-353 min-max 350
minimum-size 581-582
algorithm 581-582
min-max $581-582$
minimum-weight 581
min-max 581
simple $b$-edge cover polytope $\mathbf{3 5 0}, \mathbf{5 8 1}$
bipartite 350
simple $b$-matching 339-343, 354, 358, 569-574, 582
bipartite 339-343, 354, 358
maximum-size $339,342-343$, 358
algorithm $342-343$
complexity 358
min-max 339
maximum-weight 340-343
algorithm $342-343$
$\min -\max \quad 340-341$
maximum-size $569,572-573,582$
algorithm 572
$\min -\max 569$
maximum-weight 571-573
algorithm 571-572
min-max 571
simple $b$-matching, perfect $\sim \equiv$ $b$-factor
simple $b$-matching polytope $\mathbf{3 4 0}, \mathbf{5 7 0}$ 571, 574
adjacency 574
bipartite 340
facets 574
simple barrier $\mathbf{6 2 4}$
simple closed curve 1321, 1352
simple digraph 29
simple graph $\mathbf{1 6}$
simple $k$-edge cover $\mathbf{5 8 2}$
minimum-size 582 min-max 582
simple $k$-matching 572
maximum-size 572
min-max 572
simple $k$-matching, perfect $\sim \equiv$ $k$-factor
simple perfect 2 -matching $\equiv 2$-factor simple perfect $b$-matching $\equiv b$-factor simple perfect $k$-matching $\equiv k$-factor simple vector 11, 339, 349
simplex method $67-68,118,162-164$, 167, 195-196, 290-291, 295, 297298, 300, 344, 361, 367, 372, 374-375, 460, 561, 984, 1003, 1054, 1245-1248, 1250
simplicial entry of matrix 1444
simplicial vertex $\mathbf{1 1 3 9}$
sink 30
sink-optimal 162
size
of data 38
of fractional $c$-covering $\mathbf{3 7}$
of fractional $c$-packing $\mathbf{3 6}$
of fractional covering $\mathbf{3 7}$
of fractional packing $\mathbf{3 6}$
of fractional stable set 1090
of linear inequality $\mathbf{6 8}$
of rational number 38,68
of vector $\mathbf{1 1}, \mathbf{2 8 6}, \mathbf{3 1 8}, \mathbf{5 2 0}, \mathbf{5 3 1}$, 546, 1378, 1429
of word 40
size of vector, input $\sim \mathbf{6 9}$
skeleton, 1-~ 65
skew partition 1112
skew-symmetric matrix 429
solution, feasible $\sim 14,63$
solution, optimum $\sim 14,63$
solvable in linear time, problem $\sim \mathbf{4 7}$
solvable in polynomial time, problem $\sim$ 39-40
solvable in strongly polynomial time, problem $\sim 47$
solvable system of linear inequalities 61
source 30
source-optimal 162
source-sink connected digraph 964967, 972-976
span function 666-667
spanned by face $\mathbf{1 2 3 6}$
spanning set 693
minimum-weight 693
min-max 693
spanning set, common $\sim$ 701, 716, 741
minimum-size 701
min-max 701
minimum-weight 716 min-max 716
spanning set of matroid $\mathbf{6 5 1}$
spanning set polytope 692-693, 730, 734
spanning set polytope, common $\sim$ 715-716
spanning sets, disjoint common $\sim 741$
min-max 741
spanning subgraph 18,30
spanning tree 22, 251-252, 855-860, 862-866, 868-869, 871-876
minimum-requirement 251-252
shortest 855-860, 862-866, 868869, 871-876
algorithm 856-860
complexity 864-865
history 871-876
min-max $\quad 862-863$
uniqueness 868-869
spanning tree polytope $861-862,882-$ 885
facets 862
spanning trees, disjoint $\sim$ 877-880, 888-892, 1456
algorithm 879-880, 888-889
complexity 889-890
fractional 891 complexity 891
min-max 877-878
spanning vector $\mathbf{7 7 5}$
spans arc, set $\sim 29$
spans edge, set $\sim \mathbf{1 7}$
special 2-join 1114
splaying 271
split graph 1141
splits a set, set ~ 792, 1040
splits vertex pair, set $\sim \mathbf{1 2 6 7}$
splittable vertex 1210
splitting component 469
splitting of graph 1239
square 1121
square-free 2-matching $\mathbf{3 4 1}$
square-free graph 1121

SRR $\equiv$ system of restricted representatives
stable matching 311-314
bipartite 311-314
algorithm 312-314
maximum-weight 313-314
algorithm 313-314
stable matching polytope 312-313
bipartite 312-313
stable set 23, 315-317, 348, 352, 361, 536-539, 972, 1023, 1083-1085, 1095, 1098-1199, 1208-1217
bipartite 316-317, 348, 352, 361, 972, 1023, 1135
maximum-size 316-317, 972, 1023, 1135
algorithm 316
min-max 317
maximum-weight $348,352,361$
algorithm 352, 361
min-max 348
in claw-free graph 1208-1216
maximum-size 1208-1212
algorithm 1208-1212
maximum-weight 1213-1216 algorithm 1213-1216
in digraph 1131
in hypergraph 1428
in perfect graph 1106-1134, 11531157, 1159
maximum-size 1106-1134,
1153-1154
algorithm 1153-1154
maximum-weight 1155-1157, 1159
algorithm 1155-1157, 1159
in t-perfect graph 1186-1195
maximum-weight 1186-1195 algorithm 1186-1187
maximum-size 315-316, 536-539, 1084-1085, 1095, 1098-1185, 1196-1199, 1208-1212, 1217
NP-completeness 1084-1085, 1217
maximum-weight 1099-1101, 11551157, 1186-1195, 1213-1216
stable set, 2-~ 531-532, 578, 1091
maximum-size 531-532
algorithm 532
min-max 532
maximum-weight 578, 1091
algorithm 1091 min-max 578
stable set, extreme $\sim 1213$
stable set, $F$-~ 1203
stable set, fractional ~ 532-533,
1090-1093, 1095-1096, 1099
in hypergraph 1429
maximum-weight 1091
algorithm 1091
stable set, $k$-~
in hypergraph 1429
stable set, strong fractional $\sim 1096$, 1098-1099
maximum-size 1096
stable set, $w$-~ $\mathbf{3 1 8}, \mathbf{3 4 7}-349,534$, 578, 1200-1201
bipartite 318, 347-349
maximum-size 318
$\min -\max 318$
maximum-weight 348
min-max 348
even $w \quad$ 534, 578 maximum-size 534
min-max 534
maximum-weight 578 $\min -\max 578$
maximum-size 534
maximum-weight 1200-1201
stable set number 23, 315-317, 1083
stable set number, fractional $\sim 533$, 1090
stable set number, strong fractional ~ 1096
stable set of pairs $\mathbf{1 0 3 2}$
stable set polyhedron, 2-~ 1091
stable set polyhedron, $w$-~ 349, 1200-1201
bipartite 349
stable set polytope 319, 1088-1090, 1104, 1111, 1119-1120, 11861195, 1216, 1348-1350, 14571458
adjacency 1089-1090
bipartite 319
facets 1088-1089, 1216
of claw-free graph 1216
of perfect graph 1111
of t-perfect graph 1186-1195
stable set polytope, fractional ~
1090-1093
vertices 1091-1092
stable set problem 1084
star 21, 24
starting arc of walk 31
starting edge of walk 19
starting vertex of walk 19,31
Steiner network problem 991
Steinitz' exchange property 654, 676
step $\mathbf{7 2 2}$
step in algorithm $\mathbf{3 9}$
straight decomposition 1355
straight-line planar graph 1367
strategy, column ~ 296
strategy, row $\sim 296$
strength $\mathbf{8 7 8}, 891$
strict gammoid 659-661
strong Chvátal rank 608
strong component 90, 94-95
algorithm 90
complexity 94-95
strong component of digraph 32
strong connectivity augmentation 969973
algorithm 971-972
strong connectivity augmentation problem 969
strong connector 969-980, 1024
minimum-size 972
min-max 972
shortest 969-973, 1024
algorithm 971-972
min-max 971-972
strong connectors, disjoint $\sim$ 973-976
algorithm 975-976
min-max 973-974
strong fractional stable set 1096, 1098-1099
maximum-size 1096
strong fractional stable set number 1096
strong perfect graph conjecture 1107, 1123-1124, 1178-1181, 11841185
strong perfect graph theorem 1085, 1107, 1116, 1120-1127, 1145
strong product of graphs $\mathbf{1 1 6 7}$
strongly base orderable matroid 738743
strongly bipartite graph 1328, 13331334, 1414
strongly bipartite signed graph 1330 1333
characterization 1333
strongly chordal graph 1142
strongly connected component 90, 9495
algorithm 90
complexity 94-95
strongly connected component of digraph 32
strongly connected digraph 32, 93
strongly connected orientation 10371040, 1048
algorithm 1037-1038
characterization 1037-1040
strongly $k$-connected digraph 238, 1051
minimum-size 1051
strongly $k$-connected orientation 1044 1046
algorithm 1045
characterization 1044-1046
strongly perfect graph 1144-11451146
strongly polynomial time, problem solvable in $\sim \mathbf{4 7}$
strongly polynomial-time algorithm 47-48, 69-70
strongly polynomial-time algorithm, semi-~ 48
strongly polynomial-time solvable problem 47
strongly t-perfect graph 1187-1195, 1458
subdivision 25
subdivision, $H-\sim \mathbf{2 5}$
subgraph
of digraph $\mathbf{3 0}$
of graph 18
of signed graph 1330
subgraph, $H \sim 18$
subgraph, induced $\sim 18,30$
subgraph, proper $\sim \mathbf{1 8}, 30$
subgraph, spanning $\sim \mathbf{1 8}, \mathbf{3 0}$
subgraph induced by 18,30
subgraph with prescribed degrees 586
subgraphs, arc-disjoint $\sim \mathbf{3 0}$
subgraphs, disjoint $\sim \mathbf{1 8}, 30$
subgraphs, edge-disjoint $\sim 18$
subgraphs, vertex-disjoint $\sim \mathbf{1 8}, \mathbf{3 0}$
subgraphs with prescribed degrees 588, 591-593
subhypergraph, partial $\sim 1437,1439$
subject to capacity, flow $\sim \mathbf{1 4 8}$
subject to capacity, multiflow $\sim$ 1221-1222
submodular flow 1018-1021, 1034
minimum-cost 1019-1020, 1034 algorithm 1019-1020, 1034 min-max 1019
submodular flow polyhedron 1018, 1034
dimension 1034
facets 1034
submodular function 665, 766-826852, 1018-1034
operations on 781-782
submodular function, crossing $\sim \mathbf{8 3 8}$, 1018
submodular function, generalized $\sim$ 851
submodular function, intersecting $\sim$ 832
submodular function, symmetric $\sim$ 792
submodular function minimization 786-794
algorithm 786-792
complexity 791-792
submodular function minimization, odd

$$
\sim 793-794,842-845
$$

algorithm $793-794,842-845$
submodular function minimization, symmetric $\sim 792-793$
algorithm 792-793
submodular on crossing pairs, function $\sim 838,1018$
submodular on intersecting pairs, function $\sim \mathbf{8 3 2}$
subpartition 908, 929
subpermutation matrix 311
substar 323, 477
substar polytope $\mathbf{3 2 3}$
subtour elimination constraint 984985
subtree $\mathbf{2 2}$
subtree diameter 770
subtrees of tree 1142-1143
sum of graphs, $k-\sim \mathbf{2 6}$
sums of circuits 493-498, 1424-1426
in matroid 1424-1426
sums of circuits property $\mathbf{1 4 2 4}$
supermodular colouring 849-851, 943
supermodular function 766, 774-775, 1022-1023
supermodular function, crossing $\sim$ 1022
supermodular function, intersecting $\sim$ 837
supermodular on crossing pairs, function ~ $\mathbf{1 0 2 2}$
supermodular on intersecting pairs, function ~ 837
superorientation 1126
superperfect graph $\mathbf{1 1 5 1}$
superstar 325
superstar polytope $\mathbf{3 2 5}$
supply digraph 1221
supply graph 1222
support of vector $\mathbf{1 1}$
supporting hyperplane 63
surface 1316-1317, 1352-1371
surface, graph on $\sim 1316-1317,1352-$ 1371
surjection 13
survivable network design problem 991
swap 98
switchbox 1324
switchbox, generalized $\sim 1324$
Sylvester's graph $\mathbf{4 3 4}$
symmetric chain 236
symmetric collection $\mathbf{8 4 5}$
symmetric difference $\mathbf{9}$
symmetric digraph 1131
symmetric partially ordered set $\mathbf{2 3 6}$
symmetric set function $\mathbf{7 9 2}$
symmetric submodular function $\mathbf{7 9 2}$
symmetric submodular function minimization 792-793
algorithm 792-793
symmetric traveling salesman polytope 983-991, 995-996, 1457
adjacency 990
diameter 990,1457
dimension 990
facets 985, 987-988
symmetric traveling salesman problem
981-991, 995-1004
Christofides' heuristic 989
Lin-Kernighan heuristic 996
nearest neighbour heuristic $\mathbf{9 9 5}$
NP-completeness 982
synthesis, network ~ 1049-1057
synthesis problem, network $\sim 1051$
system 9
system of distinct representatives $\equiv$ transversal
system of restricted representatives

$$
388,407
$$

characterization 388
system of restricted representatives, common $\sim 407$
characterization 407
$T$-border 501
$T$-border, reduced $\sim 507$
$T$-cut $488-519,1413,1417-1418$
minimum-capacity $498-500,507-$ 510
algorithm 499-500
minimum-size $499,507-508,1413$ min-max 499, 507-508
$T$-cut polytope 498-499, 507-510
$T$-cuts, disjoint ~ 488-490, 501-507, 518, 1413, 1417-1418
complexity 518
min-max 489-490
$T$-join 485-519, 1417-1418
minimum-size $\quad 488-490,502,504$ $\min -\max \quad 489-490,502,504$
shortest 485-486, 488-491, 501507, 517-518
algorithm 485-486
complexity 486,518
min-max 491
$T$-join polytope $\mathbf{4 9 0}-492,501-507,517$
adjacency 517
diameter 517
$T$-joins, disjoint $\sim 507-510,519,1413$, 1456
min-max $507-508$
$T$-path 1279, 1289
$T$-path, weak ~ 1289
$T$-paths, disjoint $\sim$ 1279-1295
algorithm 1283-1284
min-max 1279-1280
$T$-paths, edge-disjoint $\sim 1282-1283$, 1285-1286
algorithm 1285-1286
min-max 1282-1283
$T$-paths, internally disjoint $\sim 1282$
min-max 1282
$T$-paths, internally vertex-disjoint $\sim$ 1282
$\min -\max 1282$
$T$-paths, vertex-disjoint $\sim 1279-1280$, 1283-1284
algorithm 1283-1284
min-max 1279-1280
$T$-paths theorem, Gallai's disjoint $\sim$ 1279-1280
$T$-paths theorem, Mader's edge-disjoint ~ 1282-1283, 1289
$T$-paths theorem, Mader's internally disjoint ~ 1282
t-perfect graph 1099, 1186-1195, 1207, 1349-1350, $\mathbf{1 4 5 8}$
t-perfect graph, strongly $\sim 1187$ 1195, 1458
table set, strong fractional $\mathrm{s} \sim \mathbf{1 0 9 6}$, 1098-1099
maximum-size 1096
tail of arc 29
TDI $\equiv$ totally dual integral
TDI, box- $\sim \equiv$ box-totally dual integral
tensor product
of matrices 12,1168
of vectors $\mathbf{1 2}, 1161$
tentative distance 97
terminal 1221-1222, 1268
$\mathrm{TH}(G) \quad 1161-1166,1169,1176,1350$
threshold graph 1141
tight constraint $\mathbf{6 4}$
tight cut 609, 619
tight inequality $\mathbf{6 3}$
tight subset $\mathbf{3 7 9}, \mathbf{1 2 6 7}, 1297$, 1310 1311
tooth $\mathbf{9 8 7}$
topological graph $\mathbf{2 5}$
topological order $\mathbf{8 9 - 9 0}$ algorithm 90
topological order, pre-~ 89-90 algorithm 89-90
total colouring 482, 1455-1456
total colouring number $\mathbf{4 8 2}$
total degree of vertex of graph 518
total order $\mathbf{1 1}$
total value
of collection of $T$-borders $\mathbf{5 0 2}$
of multiflow 1221-1222
totally balanced bipartite graph 1444
totally balanced hypergraph 1446 1447
totally balanced matrix $\quad 1444-1447$
totally dual half-integral $\mathbf{8 1}$
totally dual integral 76-83
totally dual integral, box-~ $\mathbf{8 3}$
totally dual integral, minimal $\sim 82$
totally dual quarter-integral $\mathbf{8 1}$
totally odd $K_{4}$-subdivision 1196
totally unimodular matrix $\quad \mathbf{7 5 - 7 6 , ~} 82$
tournament $\mathbf{3 0}$
transitive closure 94
transitive graph, vertex-~ $\mathbf{1 1 6 9}$
transportation $\mathbf{3 4 4}-346,356-357$, 361-377
history 362-377
minimum-cost $344-346,356-357$, 361-377
algorithm 344-346
min-max 345
transportation, $b-\sim$ 343-346, 356-357, 361-377
minimum-cost $344-346,356-357$, 361-377
algorithm 344-346 complexity 356-357
transportation, capacitated $\sim 357-$ 358, 361-377
minimum-cost $357-358,361-377$ complexity 357-358
transportation, capacitated $b-\sim 357-$ 358, 361-377
minimum-cost $\quad 357-358,361-377$ complexity 357-358
transportation, history of $\sim 362-377$
transportation polyhedron, dual $\sim$ 347
diameter 347
dimension 347
vertices 347
transportation polytope 346-347
dimension 346-347
transportation problem $\mathbf{3 4 4}$
transportation problem, Hitchcock-Koopmans ~ $\mathbf{3 4 4}$
transshipment 173-176, 182-183, 186189, 191-192, 207-210, 345-346,
362-377
algorithm 176
characterization 174-175
history 362-377
minimum-cost 182-183, 186-189, 191-192, 345-346
algorithm 182-183, 186-189
complexity 191
min-max 191-192
transshipment, b-~ 173-175, 182-184, 186-189, 191-192, 345-346
characterization 174-175
minimum-cost 182-183, 186-189, 191-192, 345-346
algorithm 182-183, 186-189
complexity 191
min-max 191-192
transshipment, history of $\sim 362-377$
transshipment polytope, $b-\sim$ 207-210
transshipment space, $b$-~ 208
dimension 208
transversal 378-392
algorithm 379
characterization 379
exchange property 381, 386-387
history 390-392
minimum-weight 382-383
algorithm 382
min-max 382-383
transversal, capacitated common $\sim$ 407
transversal, common ~ 393-409, 703
algorithm 394
characterization 393-394
exchange property 407-408
minimum-weight 395-397
algorithm 396
min-max 396-397
NP-completeness 408
transversal, common partial ~393395, 397-399
maximum-size 394 min-max 394
maximum-weight 397-399 algorithm 397 min-max 398-399
transversal, independent $\sim 702$
characterization 702
transversal, partial ~ 379-380-383
maximum-size 379-381 min-max 379-381
maximum-weight 382-383 algorithm 382 min-max 383
transversal matroid 658-659, 727-728, 739
transversal polymatroid 785
transversal polytope 384-385
transversal polytope, common $\sim 401$ 402
transversal polytope, common partial $\sim$ 399-400
transversal polytope, partial ~383385
transversals, covering by common $\sim$ 405-406
min-max 405-406
transversals, covering by common partial ~ 402-403, 406
min-max 402
transversals, covering by partial $\sim$ 386-387
min-max 386
transversals, disjoint ~ 385-386, 728
min-max 385,728
transversals, disjoint common $\sim 402-$ 405
min-max 402-403
traveling salesman polytope 983-992996, 1003
traveling salesman polytope, asymmetric ~ 992-996, 1003
adjacency 994
diameter 994
dimension 994
facets 992
traveling salesman polytope, monotone
$\sim 991$
traveling salesman polytope, symmetric ~ 983-991, 995-996, 1457
adjacency 990
diameter 990, 1457
dimension 990
facets 985, 987-988
traveling salesman problem 981-1004, 1457
history 996-1004
NP-completeness 982
traveling salesman problem, asymmetric ~ 981-982, 992-1004
NP-completeness 982
traveling salesman problem, Euclidean $\sim \mathbf{9 8 2}, 990$
traveling salesman problem, symmetric ~ 981-991, 995-1004
Christofides' heuristic 989
Lin-Kernighan heuristic 996
nearest neighbour heuristic $\mathbf{9 9 5}$
NP-completeness 982
traveling salesman tour $\mathbf{9 8 2}$
traverse 19, 31
tree 22, 855
tree, 1-~ 985-986
shortest 985
tree, directed 1-~ 993
shortest 993
tree, directed $\sim \mathbf{3 4}$
tree, Gomory-Hu ~ 248-253
algorithm 250-251
complexity 251
tree, rooted $\sim \mathbf{3 4}$
tree, shortest $\sim$ see shortest spanning tree
tree, spanning $\sim \mathbf{2 2}, 251-252,855-$ 860, 862-866, 868-869, 871-876
minimum-requirement 251-252
shortest 855-860, 862-866, 868869, 871-876
algorithm 856-860
complexity 864-865
history 871-876
min-max $862-863$
uniqueness 868-869
tree-growing method 856-858, 871873, 875
tree-hypergraph 1446
tree polytope, spanning $\sim \mathbf{8 6 1}-862$, 882-885
facets 862
tree-representation 215
tree-representation, rooted $\sim \mathbf{2 1 5}$
trees, disjoint $\sim 1242,1322,1325,1371$
complexity 1325
planar 1242 algorithm 1242
trees, disjoint spanning $\sim$ 877-880, 888-892, 1456
algorithm 879-880, 888-889
complexity 889-890
fractional 891 complexity 891
min-max 877-878
trees problem, vertex-disjoint $\sim \mathbf{1 2 4 2}$, 1322
trees theorem, Tutte-Nash-Williams' disjoint ~877-878, 931, 1048
triangle 20,539
triangle cluster $\mathbf{5 4 2}$
triangle-free 2-matching 539-544
maximum-size $542-544$
triangle-free 2-matching polytope 539-544
facets 544
triangle-free perfect 2-matching 544 algorithm 544
triangle inequality $\mathbf{9 8 2}, \mathbf{9 8 9}$
triangulated graph $\equiv$ chordal graph
trivial cut 619
trivially perfect graph $\mathbf{1 1 4 1}$
truncation, Dilworth ~820-821-825
truncation of matroid, $k-\sim 654$
TSP $\equiv$ traveling salesman problem 981-1004
Tutte-Berge formula 413-414, 440442, 723
Tutte matrix 429-430
Tutte-Nash-Williams' disjoint trees theorem 877-878, 931, 1048
Tutte's 1-factor theorem 414-415, 425, 435-436
two-processor 428
two-processor scheduling 428-429
unbounded face of planar graph $\mathbf{2 6}$
under capacity, flow $\sim 148$
underlying signed graph of bidirected graph 1201
underlying undirected graph
of bidirected graph 1201
of directed graph 29
of signed graph 1329
undirected circuit $\mathbf{3 2}$
undirected graph 16
undirected graph, underlying $\sim$
of bidirected graph 1201
of directed graph 29
of signed graph 1329
undirected Hamiltonian circuit 115
NP-completeness 115
undirected Hamiltonian path problem 114-115
NP-completeness 114-115
undirected $k$-commodity flow problem 1222
undirected maximum-value $k$-commodity flow problem 1222
undirected maximum-value multiflow problem 1222
undirected multiflow problem 1222
undirected walk
in digraph 31
uniform hypergraph 1381
uniform hypergraph, $k$-~ 36, 755
uniform matroid 654
uniform matroid, $k$-~ $\mathbf{6 5 4}$
unimodular graph 1147
unimodular hypergraph 1448-1451
characterization 1448-1449
unimodular matrix, totally $\sim$ 75-76, 82
union, matroid $\sim$ 725-744
history 743-744
union of antichains 226, 235, 1027
min-max 226
union of arborescences 916-918
union of branchings 915-918
min-max 916-918
union of chains 228-229, 1026-1027 min-max 228-229
union of directed cuts 224-226 acyclic 224-226
union of disjoint edge covers 350 bipartite 350
min-max 350
union of disjoint $s-t$ cuts 211-212
algorithm 212
min-max 211-212
union of forests 877,890
maximum-size 890
complexity 890
maximum-weight 890
complexity 890
min-max 877
union of independent sets 726
matroid union theorem 726
min-max 726
union of matchings 340
bipartite 340
min-max 340
union of matroids $\mathbf{7 2 6}$
union of $r$-arborescences 913-915
min-max 913,915
union of $s-t$ paths 210-213, 227-228
algorithm 212
complexity 212
min-max 210-211
minimum-cost 212-213
complexity 212-213
union theorem, matroid $\sim$ 726, 782
unit base vector 12
unsplittable flow 196
up hull 59
up-monotone ideal 11
up-monotone subset of $\mathbb{R}^{n} \quad 65$
upper ideal 11, 1028
upper semimodular lattice 669, 675, 677, 681-682
valent vertex, $k$-~ $\mathbf{1 7}$
valid inequality $\mathbf{6 0}$
value
of flow 148
of homotopic circulation 1360
of multiflow 1221-1222
of $T$-border 502
value, total ~
of collection of $T$-borders 502
of multiflow 1221-1222
value giving oracle 771
valued vector, $\{0,1\}-\sim \mathbf{1 1}$
variable 44
vector, $0,1 \sim \mathbf{1 1}$
vector, $\{0,1\}$-valued $\sim \mathbf{1 1}$
vector, integer $\sim \mathbf{1 1}, \mathbf{7 3}$
vertex
of digraph 28
of graph 16
of hypergraph $\mathbf{3 6}, 1375$
of polyhedron
64-65
vertex-colourable graph, $k$-~ 23, 1083
vertex-colouring 23, 1083-1088, 1098, 1101-1185, 1206-1207
vertex-colouring, $k$-~ 1083
vertex-colouring, minimum $\sim 23$, 1083-1088, 1098, 1102-1185, 1206-1207
NP-completeness 1084-1085
of perfect graph 1106-1134, 11541155
algorithm 1154-1155
vertex-colouring, minimum fractional $\sim$ 1096, 1098
vertex-colouring, minimum weighted $\sim$ 1096-1097, 1157-1159
NP-completeness 1096-1097
of perfect graph 1157-1159 algorithm 1157-1159
vertex-colouring number 23, 1083
vertex-connected digraph, $k$-~ 238, 1050-1051
minimum-size 1050-1051
vertex-connected graph, $k$-~ 237, 1049-1050
minimum-size 1049-1050
vertex-connectivity 237-238-243, 253-255, 1049-1051, 1074-1078, 1458
algorithm 239-241
complexity 241
vertex-connectivity, 2-~ 243
algorithm 243
vertex-connector, 2-~ 1077-1078
minimum-size 1077-1078 min-max 1077-1078
vertex-connector, $k$-~ 1074-1075, 1077
minimum-size 1074-1075 min-max 1074-1075
vertex cover 23, 260-263, 265, 277, 304-305, 315-316, 338, 343, 536539, 1083-1085, 1095, 1103-
$1105,1136,1159,1175,1187$, 1199-1200
bipartite 260-263, 265, 277, 304305, 338, 343
minimum-size 260-262, 265, 277, 304-305
algorithm 265
complexity 277
min-max 260-261
minimum-weight 338, 343
algorithm 343
min-max 338
in hypergraph 1377, 1380-1381
minimum-size 1380-1381
in perfect graph 1159
minimum-weight 1159
algorithm 1159
in t-perfect graph 1187
minimum-weight 1187
minimum-size $315-316,536-539$, 1084-1085, 1095, 1103-1105, $1136,1175,1199-1200$
bipartite 1136
NP-completeness 1084-1085
minimum-weight 1187
vertex cover, 2-~ 520-521, 531-532, 556-557, $\mathbf{1 0 9 4}$
minimum-size $520-521,531-532$
algorithm 521
min-max 520-521
minimum-weight 556-557, 1094
algorithm 1094
min-max 557
vertex cover, fractional ~ 521, 1093 1095
in hypergraph 1378, 1380-1381
minimum-size 1380-1381
minimum-weight 1094
algorithm 1094
vertex cover, $k$-~
in hypergraph 1378
vertex cover, $w-\sim$ 285-286, 289-290,
304, 337-338, 343, 523, 557-558
bipartite 285-286, 289-290, 304, 337-338, 343
minimum-size 285-286, 289290, 304
algorithm 289-290
min-max 285-286
minimum-weight 337-338 min-max 338
even $w \quad 523,558$
minimum-size 523
min-max 523
minimum-weight 558 min-max 558
minimum-size 523
minimum-weight 557-558
vertex cover number 23, 260, 315316, 1083
vertex cover number, fractional ~ 521, 1093
vertex cover polyhedron, 2-~ 1094
vertex cover polyhedron, $w$-~ 339
bipartite 339
vertex cover polytope 305,1088 , 1187, 1348-1350
bipartite 305
of t-perfect graph 1187
vertex cover polytope, fractional ~ 1094-1095
vertices 1094
vertex-cut 22, 33, 239-241, 243, 253
minimum-size 239-241
algorithm 239-241
complexity 241
vertex-cut, $k$-~ 22, 33
vertex-cut, minimum $\sim \equiv$
minimum-size vertex-cut 237-
238
vertex-cut, $S-T \sim \mathbf{2 2}, \mathbf{3 4}$
vertex-cut, $s-t \sim \mathbf{2 2}, \mathbf{3 3}, 132$
minimum-size 132
min-max 132
vertex-disjoint paths 1224-1225, 1243, 1299, 1320-1323, 1368-1370
complexity 1224-1225, 1243
planar 1299, 1320-1323, 1368-1370
algorithm 1320-1323
characterization 1320-1323
complexity 1299
vertex-disjoint paths, internally ~ 132
vertex-disjoint paths problem 1223
vertex-disjoint paths problem, $k \sim$ 1223
vertex-disjoint $\mathcal{S}$-paths 1280-1281
$\min -\max \quad 1280-1281$
vertex-disjoint $s-t$ paths, internally $\sim$ 132, 137-140, 142-147, 275-276
algorithm 137-138
complexity 139, 276
min-max 132
planar 140 complexity 140
vertex-disjoint subgraphs 18, 30
vertex-disjoint $T$-paths 1279-1280, 1283-1284
algorithm 1283-1284
min-max 1279-1280
vertex-disjoint $T$-paths, internally $\sim$ 1282
$\min -\max 1282$
vertex-disjoint trees problem 1242, 1322
vertex-disjoint walks 20, 32
vertex-disjoint walks, internally $\sim \mathbf{2 0}$, 32
vertex-transitive graph 1169
violated inequality problem, most $\sim$ 697-698, $\mathbf{7 3 3}$
Vizing's theorem 465-467-468
void 48
$w$-cover 1188
$w$-stable set 318, 347-349, 534, 578, 1200-1201
bipartite 318, 347-349 maximum-size 318 min-max 318 maximum-weight 348 min-max 348
even $w \quad 534,578$ maximum-size 534
min-max 534
maximum-weight 578
min-max 578
maximum-size 534
maximum-weight 1200-1201
$w$-stable set polyhedron 349, 12001201
bipartite 349
$w$-vertex cover 285-286, 289-290, 304, 337-338, 343, 523, 557-558
bipartite 285-286, 289-290, 304, 337-338, 343
minimum-size 285-286, 289290, 304
algorithm 289-290 min-max 285-286
minimum-weight 337-338 min-max 338
even $w \quad 523,558$
minimum-size 523
min-max 523
minimum-weight 558 min-max 558
minimum-size 523
minimum-weight 557-558
$w$-vertex cover polyhedron 339
bipartite 339
$w$-weight 13
Wagner's theorem 26-27
walk
in digraph 31
in undirected graph 19
walk, closed $\sim 20$
walk, closed directed $\sim 32$
walk, directed $\sim 31$
walk, Eulerian ~ 24
walk, Eulerian directed ~ $\mathbf{3 4}$
walk, even ~ 19
walk, odd $\sim 19,593$
walk, reverse ~ 19
walk, $S-T \sim 19,31$
walk, $S-t \sim 19$
walk, $s-T \sim 19$
walk, $s-t \sim 19,31$
walk, undirected $\sim$
in digraph 31
weak 3 -flow conjecture 473, 1454
weak component 208
weak component of digraph 32
weak duality 62
weak perfect graph conjecture 1107
weak $T$-path $\mathbf{1 2 8 9}$
weakly bipartite graph 1326-13271329, 1334-1341, 1392
weakly bipartite signed graph 1330 1331, 1340
characterization 1340
weakly chordal graph 1148
weakly connected component of digraph 32
weakly connected digraph 32
weakly polynomial-time algorithm 48
weakly triangulated graph $\equiv$ weakly chordal graph 1148
weight 13, 523, 554
weight, $w-\sim 13$
weight function 13
weighted clique cover number 1097
weighted clique cover number, fractional $\sim 1097$
weighted colouring, minimum $\sim 1096-$ 1097, 1157-1159
NP-completeness 1096-1097
of perfect graph 1157-1159 algorithm 1157-1159
weighted colouring, minimum fractional ~ 1097
NP-completeness 1097
weighted colouring number 1096
weighted colouring number, fractional ~ 1097
weighted vertex-colouring, minimum ~ 1096-1097, 1157-1159
NP-completeness 1096-1097
of perfect graph 1157-1159 algorithm 1157-1159
well-balanced orientation 1043
well-characterized 43
wheel 1194
width-length inequality 1383,1385
windy postman problem 518
Woodall's conjecture 962-964, 966968, $\mathbf{1 4 5 7}$
word 40
Young diagram 230
$\mathbb{Z}_{+}$-max-flow min-cut property 1397

