G. H. von Wright – Deontic Logic

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Introduction

- In von Wright's 1951 formulation, deontic logic is intended to modalize the obligatory, the permitted and the forbidden. NB: permission functions as a primitive in his system, and the other two categories are defined in terms of permission.
- I will explain what the deontic realm is and what deontic units of the realm are, and then illustrate their usage with a decision problem.
- Then, I discuss his claim that deontic logic is of more than logical interest, but is of philosophical interest, by examining the principles and tautologies of his system.

OVERVIEW

Deontic Logic

- The atoms of the system are individual-acts.
- The connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ are defined as normal. Molecular sentences are formed by combining atoms with connectives, and represent complex actions.
- The modal operators P and O represent "it is permitted that…" and "it is obligatory that…"
- *E.g.* $P(a \wedge b)$ indicates "It is permitted to (perform the complex action) a and b." Such sentences are "P-sentences." (Similarly for obligation sentences, or ("O-sentences".)

The Deontic Categories

- The permission operator P is primitive.
- Obligation is defined as follows: $Oa := \neg P \neg a$.
- Other categories are mentioned: *E.g.* the (non-formalized) forbidden category is defined as $\neg Pa$. Indifferent acts are are permitted and their negation is permitted, *i.e.* $Pa \wedge P\neg a$.

Perfect Disjunctive Normal Form

- The key type of molecular sentences for the purpose of deciding the truth value of a deontic proposition is the perfect disjunctive normal form [or "PDNF"].
- PDNF is a statement in DNF where each atom appears in a literal of every disjunct. Each disjunct can be thought of as a complete atomic description, called the *P*-constituent.
- Quick reminder: DNF is found by recursively changing all connectives to $\neg, \wedge, \vee$ and then making disjunctions, each of which only containing connectives $\neg, \wedge$.
- *E.g.* making PDNF:

$$
\begin{aligned}
& P(a \rightarrow (b \wedge c)) \\
\equiv \quad & P(\neg a \vee (b \wedge c)) \\
\equiv \quad & P(\neg a \wedge b \wedge c) \vee P(\neg a \wedge \neg b \wedge c) \vee P(\neg a \wedge b \wedge \neg c) \vee \\
& P(\neg a \wedge \neg b \wedge \neg c) \vee P(a \wedge b \wedge c)
\end{aligned}
$$

Deontic Realm and Deontic Units

- The purpose of generating PDNF is to determine truth-values for deontic statements from the P-constituents in PDNF.
- For given n atoms, the deontic realm is the disjunction of all possible P-sentences which are conjunctions of literals, and each of the n atoms occur in each conjunction.
- Thus, there are 2^n P-sentences for any n atoms, since every atom can either appear positively or negatively in a literal.
- Each of these P-sentences (*i.e.* each disjunct) is a deontic unit.

Example

- *E.g.* Continuing the previous example, there are $2^3 = 8$ units in the deontic realm of a, b, and c (*i.e.* from $Pa \rightarrow (b \wedge c)$), but only 5 deontic units in the PDNF of the original sentence.

$$\begin{aligned} & P(a \rightarrow (b \wedge c)) \\ \equiv \quad & P(\neg a \vee (b \wedge c)) \\ \equiv \quad & P(\neg a \wedge b \wedge c) \vee P(\neg a \wedge \neg b \wedge c) \vee P(\neg a \wedge b \wedge \neg c) \vee \\ & P(\neg a \wedge \neg b \wedge \neg c) \vee P(a \wedge b \wedge c) \end{aligned}$$

A Decision Problem

- Since all deontic modal operators can be defined in terms of permission, and permission can be defined in terms of deontic units, we can solve deontic decision problems.
- Here is an example:

$$
\begin{aligned}
& [\neg Pb \wedge O(a \rightarrow b)] \rightarrow \neg Pa \\
\equiv \quad & \neg[\neg Pb \wedge O(a \rightarrow b)] \vee \neg Pa \\
\equiv \quad & Pb \vee P\neg(\neg a \vee b) \vee \neg Pa \\
\equiv \quad & Pb \vee P(a \wedge \neg b) \vee \neg Pa
\end{aligned}
$$

A Decision Problem con't

The P-constituents of $Pb \vee P(a \wedge \neg b) \vee \neg Pa$ are (1) $P(a \wedge \neg b)$, (2) $P(a \wedge b)$, and (3) $P(\neg a \wedge b)$.

$P(a \wedge \neg b)$	$P(a \wedge b)$	$P(\neg a \wedge b)$	Pb	$P(a \wedge \neg b)$	$\neg Pa$	$\vee$
T	T	T	T	T	F	T
T	T	F	T	T	F	T
T	F	T	T	T	F	T
F	T	T	T	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	F	T	T
F	F	F	F	F	T	T

So $\neg Pb \wedge O(a \rightarrow b) \rightarrow \neg Pa$ is a tautology [$\neg Pb \wedge O(a \rightarrow b) \vDash \neg Pa$].

THE FIRST DEONTIC PRINCIPLE

- Principle of Deontic Distribution: *If an act is the disjunction of two other acts, then the proposition that the disjunction is permitted is the disjunction of the proposition that the first act is permitted and the proposition that the second act is permitted.*
- NB: this is what allows us to do the P distribution among disjuncts in PDNF.
- Allows you to distribute permission through a formula.

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- Allows you to distribute permission through a formula.
- Fairly unassuming.

THE SECOND DEONTIC PRINCIPLE

- Principle of Deontic Contingency: *A tautologous act is not necessarily obligatory, and a contradictory act is not necessarily forbidden.*
- This one is interesting as a philosophical consideration. In the article, von Wright suggests that intuitions are not clear, which is why he makes this a principle.

THE SECOND DEONTIC PRINCIPLE CON'T

- However, as Meyer and Wieringa point out (cf. §1.3), if we interpret von Wright's Old System as a standard modal logic,

  "(*OT*) $O(p \vee \neg p)$ 'Existence of an empty normative system'

  which, by the way, von Wright rejected as an axiom!"
- Small point: I dislike labeling it "Existence of an empty normative system" since I'm not sure emptiness is the best way of characterizing the property. Perhaps "Existence of tautological obligations"?
- In von Wright's defence, saying that something is "not necessarily so" is quite different from "rejecting" it.

THE SECOND DEONTIC PRINCIPLE CON'T

So here are some of the possible issues to balance when looking at the Principle of Deontic Contingency:

- The mathematical simplicity of being able to apply a Kripke-style semantics;
- Intuitions about whether all tautologies are obligated (and whether it makes sense to permit contradictory acts);
- von Wright's own goals—I have the suspicion that this principle was implemented to give a larger modeling scope. If this principle is used, then we have the flexibility to claim that a tautology is obligated (or not).

THREE TAUTOLOGIES OF COMMITMENT

- There are certain deontic tautological claims which are also tautologies in propositional logic. Such claims are trivial with respect to this logic (cf. von Wright 5). We are interested in specifically deontic claims.
- Tautologies in (i), (ii) and (iii*a*–*d*) (von Wright 13) are fairly intuitive.
- Instead, I want to examine (iii*e*–*g*).
- NB: in the paper, (iii*a*) is shown to be a tautology. In this presentation, I showed that (iii*c*) is a tautology.

THREE TAUTOLOGIES OF COMMITMENT

The following are from p. 14 of "Deontic Logic." The characterizations are his own.

iii*e*) $\neg(O(a \vee b) \wedge \neg Pa \wedge \neg Pb)$. It is logically impossible to be obliged to choose between forbidden alternatives.

iii*f*) $Oa \wedge O(a \wedge b) \rightarrow c)$ entails $O(b \rightarrow c)$. If doing two things, the first of which we ought to do, commits us to do a third thing, then doing the second thing alone commits us to do the third thing. "Our commitments are not affected by our (other) obligations."

iii*g*) $O(\neg a \rightarrow a)$ entails Oa. If failure to perform an act commits us to perform it, then this act is obligatory.

Discussion Questions

- Does the operator O more resemble $\Box$ of alethic logics or K_α of epistemic logics?
- To what extent do we view the paradoxes of Meyer and Weiringa as surprising truths or as reductios against the system?
- The suggestion at the very end of the paper that one way of relativizing the logic would be to implement different moral codes is intriguing. How could this be done? For instance, could it be that one code has a set of permissible acts, and then that you have some index for whether a certain act is permissible relative to a given code? Or do you just set up these sets and then know which group you are in (and which codes are applicable for you?