THeory and Evidence to Measure Influence in Social structures

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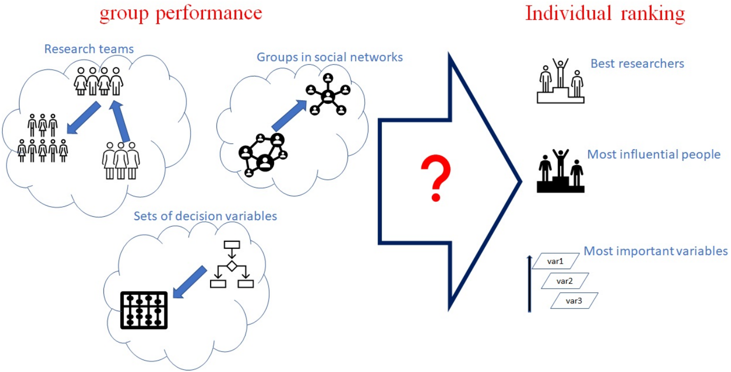
Context and motivation

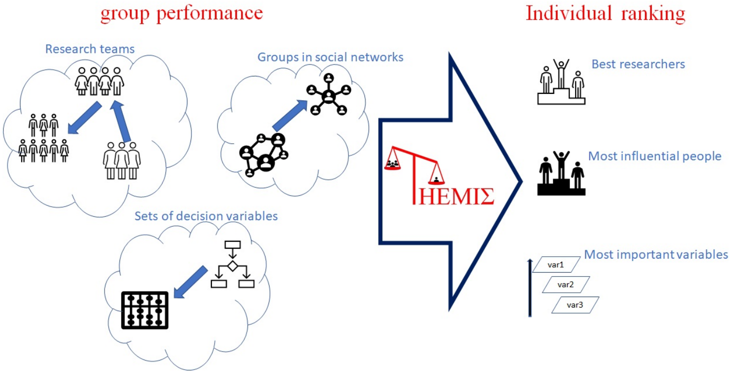
Problems apparently very different:

- the ranking of researchers or football players,
- the affectation of students to universities (in particular, in France, the Parcoursup algorithm),
- the influence of someone in a social network (like Twitter),
- the responsibility of a formula in the inconsistency of a belief base,
- the impact of some criteria in a multi-criteria decision making situation
- ...

Context and motivation



Context and motivation



Project details

- *Start Date*: March 29th (seminars announcement will follow).
- *Duration of the project*: 48 months.
- *Coordinator*: Stefano MORETTI
- *Consortium*: CRIL (Centre de Recherche en Informatique de Lens - UMR 8188, University of Artois), LAMSADE (Laboratoire d'Analyse et de Modélisation de Systèmes d'Aide la Décision - UMR 7243, Paris Dauphine University), LIP6 (Laboratoire d'Informatique de Paris 6 - UMR7606, Sorbonne University).
- Scientific Leaders: Sébastien KONIECZNY (CRIL), Paolo VIAPPIANI (LIP6) and SM (LAMSADE).
- *Members*: 14 Partners, 1 PhD, 1 Res.Eng, 14 stages.
- *Implementation*: 5 technical WPs, plus a WP Organisation and a WP Dissemination.

Basic notions and notations

A *binary relation R on a finite set X* is a subset of the Cartesian product $X \times X$. For each $x, y \in X$, the notation xRy will be preferably used instead of the more formal $(x, y) \in R$. A binary relation R is said to be:

- *reflexive*, if for each $x \in X$, xRx;
- *transitive*, if for each $x, y, z \in X$, xRy and $yRz \Rightarrow xRz$;
- *total*, if for each $x, y \in X$, $x \neq y \Rightarrow xRy$ or yRx;
- *symmetric*, if for each $x, y \in X$, $xRy \Leftrightarrow yRx$;
- *asymmetric*, if for each $x, y \in X$, $(x, y) \in R \Rightarrow (y, x) \notin R$;
- *antisymmetric*, if for each $x, y \in X$, xRy and $yRx \Rightarrow x = y$.

A reflexive, transitive and total binary relation on X is called a *total preorder* (also called, a *ranking*) on X.
An antisymmetric total preorder on X is called a *total order* on X.
$\mathcal{R}(X)$ denotes the set of rankings (or total preorders) on X.

Problem Definition

A finite set of individuals, alternatives, items...: $X = \{1, \ldots, |X|\}$

A total preorder $\succcurlyeq$ over $\mathcal{P}(X)$ (the set of all subsets of X)[1]:
$S \succcurlyeq T$: coalition $S \in \mathcal{P}(X)$ is at least as "strong" as $T \in \mathcal{P}(X)$
($\sim$ the symmetric part, $\succ$ the asymmetric part).

A *social ranking solution* $R : \boldsymbol{\mathcal{R}}(\mathcal{P}(X)) \to \boldsymbol{\mathcal{R}}(X)$
that associates to every *(coalitional) ranking* $\succcurlyeq$ over $\mathcal{P}(X)$ a total preorder $R(\succcurlyeq)$ or $R^{\succcurlyeq}$ over X.
$iR^{\succcurlyeq}j$: item $i \in X$ is at least as "relevant" as item $j \in X$
($I^{\succcurlyeq}$ the symmetric part of $R^{\succcurlyeq}$, and $P^{\succcurlyeq}$ its asymmetric part).

[1] If not specified, the empty set is considered as the worst set

Related literature

A set of "reasonable" properties that a social ranking solution $R : \boldsymbol{\mathcal{R}}(\mathcal{P}(X)) \to \boldsymbol{\mathcal{R}}(X)$ should satisfy: the lex-cel solution (Bernardi, Lucchetti, Moretti *Soc Choice and Welfare*,2019)

Other solutions: CP-majority (Haret, Khani, Moretti, Öztürk *IJCAI2018*) and ordinal Banzhaf (Khani, Moretti, Öztürk, *IJCAI2019*); cardinality-based lex-cel (Algaba, Moretti, Rémila, Solal, 2021, submitted)

Manipulability of social rankings (Allouche, Escoffier, Moretti Öztürk, *IJCAI2020*)

Other partial answer using invariant power indices (Moretti *Homo Oecon*, 2015)

Some further notations

Suppose we have a ranking $\succcurlyeq \in \mathcal{R}(\mathcal{P}(X))$ of the form

$$S_1 \succcurlyeq S_2 \succcurlyeq S_3 \succcurlyeq \cdots \succcurlyeq S_{2^{|X|}-1}.$$

Given this ranking $\succcurlyeq$, we also consider its *quotient order*, denoted as follows

$$\Sigma_1 \succ \Sigma_2 \succ \Sigma_3 \succ \cdots \succ \Sigma_l$$

in which the subsets S_j are grouped in the *equivalence classes* Σ_k generated by the symmetric part of $\succcurlyeq$.

This means that all the sets in Σ_1 are indifferent to S_1 and are strictly better than the sets in Σ_2 and so on.

Some further notations (follows)

For any element $x \in X$, denote by x_k the number of sets containing x in the indifference class Σ_k, that is

$$x_k = |\{S \in \Sigma_k : x \in S\}|$$

for $k = 1, \ldots, l$. Let $\theta_{\succeq}(x)$ be the l-dimensional vector $\theta_{\succeq}(x) = (x_1, \ldots, x_l)$ associated to $\succeq$.

Now consider the lexicographic order among vectors:

$\mathbf{x} \geq_L \mathbf{y}$ if either $\mathbf{x} = \mathbf{y}$ or $\exists j : x_i = y_i, i = 1, \ldots, j-1 \wedge x_j > y_j$.

Definition

The *lexicographic excellence (lex-cel) solution* is the function $R_{\boldsymbol{le}} : \mathcal{R}(\mathcal{P}(X)) \to \mathcal{R}(X)$ defined for any ranking $\succcurlyeq \in \mathcal{R}(\mathcal{P}(X))$ as

$$x R_{\boldsymbol{le}}(\succcurlyeq) y \qquad \text{if} \qquad \theta_{\succcurlyeq}(x) \;\; \geq_L \;\; \theta_{\succcurlyeq}(y).$$

(from Algaba, Moretti, Remila, Solal (2020))

Analyse the performance of four attacking players of the Paris Saint Germain (PSG) team during the eight matches of Champions League played during the season 2019/2020 (before the break on March 2020 for the covid-19 emergency).

It is well known that the PSG coach Thomas Tuchel has to face a selection dilemma when he must select among the four attacking stars Di María (D), Icardi (I), Mbappé (M) and Neymar (N).

We considered all different subsets of the four stars, and we assessed some relevant parameters like the total number of points scored p, the number of goals scored s and the one of goals conceded c by those groups when employed together in a match.

coalitions	points (p)	goals scored (s)	goals conceded (c)
$\{I, D, M\}$	6	6	0
$\{I, D\}$	6	4	0
$\{I, M, N\}$	3	5	0
$\{D, N\}$	3	2	0
$\{M\}$	1	2	2
$\{N, M\}$	0	1	2

A coalitional ranking has been computed according to a lexicographic comparison of vectors (p, s, c)

$$\{I, D, M\} \succ \{I, D\} \succ \{I, M, N\} \succ \{D, N\} \succ \{M\} \succ \{N, M\} \succ S,$$

for each other $S \subseteq \{D, I, M, N\}$ (which are all in the same worst equivalence class).

Lex-cel ranking

$$\{I, D, M\} \succ \{I, D\} \succ \{I, M, N\} \succ \{D, N\} \succ \{M\} \succ \{N, M\} \succ S,$$

for each other $S \subseteq \{D, I, M, N\}$ (which are all in the same worst equivalence class).

Σ_k	$\{I, D, M\}$	$\{I, D\}$	$\{I, M, N\}$	$\{D, N\}$	$\{M\}$	$\{N, M\}$	other S
$\theta_{\succeq}(D)$	1	1	0	1	0	0	5
$\theta_{\succeq}(I)$	1	1	1	0	0	0	5
$\theta_{\succeq}(M)$	1	0	1	0	1	1	4
$\theta_{\succeq}(N)$	0	0	1	1	0	1	5

So, according to the lex-cel solution
Icardi $P^{\succeq}_{le}$ Di María $P^{\succeq}_{le}$ Mbappé $P^{\succeq}_{le}$ Neymar.

CP-majority (HKMO-IJCAI2018)

We compare two individuals $i, j \in X$ based on their relative contribution to groups of other individuals: $S \cup \{i\}$ vs. $S \cup \{j\}$ for all $S \subseteq X \setminus \{i, j\}$.

$$\{I, D, M\} \succ \{I, D\} \succ \{I, M, N\} \succ \{D, N\} \succ \{M\} \succ \{N, M\} \succ S,$$

for each other $S \subseteq \{D, I, M, N\}$ (which are all in the same worst equivalence class).

I vs. D	I vs. M	I vs. N	D vs. M
$I \sim D$	$I \prec M$	$I \sim N$	$D \prec M$
$IM \sim DM$	$DI \succ MD$	$DI \succ ND$	$DI \succ IM$
$IN \prec DN$	$IN \prec MN$	$IM \sim NM$	$DN \succ MN$
$IMN \succ DMN$	$DIN \sim DMN$	$DIM \succ NDM$	$DIN \prec IMN$
$I \ I^{\succcurlyeq}_{CP} \ D$	$M \ P^{\succcurlyeq}_{CP} \ I$	$I \ P^{\succcurlyeq}_{CP} \ N$	$D \ I^{\succcurlyeq}_{CP} \ M$

Not transitive!

Ordinal Banzhaf (KMO-IJCAI2019

$$\{I, D, M\} \succ \{I, D\} \succ \{I, M, N\} \succ \{D, N\} \succ \{M\} \succ \{N, M\} \succ S,$$

for each other $S \subseteq \{D, I, M, N\}$ (which are all in the same worst equivalence class).

I vs. D	w_{ID}^S	I vs. M	w_{IM}^S	I vs. N	w_{IN}^S	D vs. M	w_{DM}^S
$I \sim D$		$I \prec M$	2	$I \sim N$		$D \prec M$	2
$IM \sim DM$		$DI \succ MD$	1	$DI \succ ND$	0	$DI \succ IM$	1
$IN \prec DN$	2	$IN \prec MN$	1	$IM \sim NM$		$DN \succ MN$	2
$IMN \succ DMN$	2	$DIN \sim DMN$		$DIM \succ NDM$	2	$DIN \prec IMN$	2
	$I \; I_W^{\succsim} \; D$		$M \; P_W^{\succsim} \; I$		$I \; P_W^{\succsim} \; N$		$M \; P_W^{\succsim} \; D$

Compute the weight w_{ij}^S of the CP-comparison on $S \subseteq X \setminus \{i, j\}$ as the number of coalitions in $\{S, S \cup \{i, j\}\}$ between $S \cup \{i\}$ and $S \cup \{j\}$, or the sum of the "ordinal" marginal contributions

So, according to the ordinal Banzhaf solution

Mbappé $P_W^{\succsim}$ Di María $I_W^{\succsim}$ Icardi $P_W^{\succsim}$ Neymar.

Why Banzhaf? Counting the number of times $i \in X$ has a strict positive marginal contribution ($S \cup \{i\} \succ S$) minus the number of times i has strict negative marginal contribution ($S \cup \{i\} \prec S$)

Lex-cel and cardinality

In Algaba et al (2020) we extend the lex-cel solution and we axiomatically characterize two novel solutions which take into account the size of the groups.

$$\{I, D, M\} \succ \{I, D\} \succ \{I, M, N\} \succ \{D, N\} \succ \{M\} \succ \{N, M\} \succ S,$$

$$M^{\succsim, I} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad M^{\succsim, D} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$M^{\succsim, M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad M^{\succsim, N} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In this case the new solutions give the same ranking as the lex-cel.

Not only football

The the lex-cel solution has been recently used to define preferences over regulatory norms that are more “aligned” with some moral values

M. Serramia, M. Lopez-Sanchez, J. A. Rodriguez-Aguilar, A qualitative approach to composing value-aligned norm systems, in: Proceedings of the 19th International Conference on Autonomous Agents and MultiAgent Systems, 2020, pp. 1233-1241

WP1: Portfolio of solutions

- *Goal*: to design procedures aimed at ranking individuals according to how they behave in various groups, to analyse their computational features and their robustness to changes in the interaction among individuals.
- *Method*: analysis of models of coalitional interaction situations where the “intensity” of the agents’ cooperation is characterized by an “ordinal” information.

WP2: Subdomains, elicitation and manipulation

- *Goal*: to analyse the consequences of the incomparability of certain coalitions (due to incompleteness of the data, heterogeneity of criteria, lack of information, etc.) and the problem of ranking elicitation and the resistance to manipulation for solutions.
- *Method*: cooperative game theory with restrictions in the possibility to form coalitions; analysis of the minimax regret under utility uncertainty; manipulability issues from social choice theory.

WP3: Coalition formation

- *Goal*: to investigate the effect of social ranking solutions on the behaviour of individuals to form stable coalition structures according to various notions of social stability, and the impact of computational complexity on the strategic behaviour of players.

- *Method*: models based on hedonic games and algorithmic generation of optimal coalition structures; concepts from algorithmic game theory to analyse the quality of optimal coalition structures.

WP4: Compact representation

- *Goal*: merging models from the literature about compact representation of cooperative games with those about compact preference representation.
- *Method*: Compact preference representation and formulation of the social ranking problem as a combinatorial optimization problem.

WP5: Explaining ordinal influence in social AI.

- *Goal*: to apply our solutions to evaluate the influence of criteria for the selection of students in the national admissions platform Parcoursup and in the analysis of social networks.
- *Method*: ordinal representation of the effects of features attributions in classification models.

Thank you!

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