Thesis topic: Fine-Grained Questions in Structural Parameterized Complexity and Approximation

Supervisors: Michael Lampis and Florian Sikora

1 Scientific Context

The vast majority of optimization problems are NP-hard, meaning that they require exponential time to solve, under standard complexity hypotheses. Despite this unpleasant fact, such problems still need to be dealt with. Some of the main avenues that have been explored for dealing with NP-hard problems include the following:

1. **Parameterized Complexity** theory, where we focus on secondary parameters, other than the size of the input, which attempt to measure the structure (or lack thereof) of an instance. In the context of graph problems, the most well-studied examples are graph width parameters, such as treewidth. For example, a plethora of problems are known to admit algorithms with running time of the form $f(k)n^{O(1)}$ on graphs of treewidth k. Such algorithms are called fixed-parameter tractable, as they become efficient for moderate values of k.
2. **Fine-Grained Complexity** theory, where we make a precise distinction on the target running time of our algorithm. The motivation here is that, even if we know that super-polynomial time algorithm is necessary to solve a problem, it is still interesting to improve 2^n to $2^{n/2}$ or even to $2^{\sqrt{n}}$ (and similarly for the function $f(k)$ in the parameterized context). Here, it is often quite interesting to also consider the corresponding lower bound questions, which typically invoke fine-grained hypotheses, such as the ETH (Exponential Time Hypothesis) and its strong variant, the SETH.
3. **Approximation** algorithms, where we accept that obtaining an optimal solution for all inputs in a reasonable amount of time is impossible, and therefore decide to sacrifice optimality. The goal here is to design algorithms with performance guarantees, which always return a solution close to optimal while still running efficiently, ideally in polynomial time.

The objective of this thesis is to work along the three axes sketched above to develop algorithms and complexity results for structural parameterizations of hard graph problems.

2 Objectives and Methodology

The focus of this thesis will be on carefully analyzing the complexity of graph problems using notions related to graph width parameters, such as treewidth [3] and its numerous variations. Typical questions that we plan to consider follow along lines such as:

1. Fine-Grained price of generality: how does the complexity of various problems change as we consider different notions of graph structure, such as treewidth, pathwidth, or tree-depth? What problem properties allow efficient algorithms for one parameter and not the other ([1])? What is the precise complexity of each problem, assuming standard complexity hypotheses such as the ETH and SETH?

2. Approximation: for problems for which precise running time lower bounds are known, can these lower bounds be worked around if we allow algorithms that obtain approximate solutions ([4])?

3. New width parameters: what other notions of structure are relevant, besides treewidth variants? How far can we generalize state-of-the-art knowledge to new widths, such as twin-width ([2])?

4. New complexity hypotheses: many of the currently known fine-grained are based on strong complexity hypotheses, such as the SETH or the Set Cover Conjecture. Can we move to a more nuanced understanding of the interplay between these hypotheses and notions of graph width ([5])?

Although the focus of the thesis will be on big-picture questions, work will proceed by considering specific concrete problems first. The problem considered will have motivations in various application fields (e.g. network optimization), but the focus will be on theoretical aspects of their computational complexity.

3 Working Conditions and Candidate Profile

The successful doctoral candidate will be jointly supervised by Dr. Michael Lampis `https://www.lamsade.dauphine.fr/~mlampis/` and Dr. Florian Sikora `https://www.lamsade.dauphine.fr/~sikora/`. He/she will be hosted in LAMSADE, Université Paris-Dauphine, located in the (very posh) 16th district in Paris. The thesis will be financed via a ministry fellowship for a period of three years, with a travel budget for attending conferences and summer schools available from the lab. The doctoral candidate will have the option of participating in teaching duties in Dauphine (for an extra stipend) if he/she so wishes.

The successful candidate is expected to have a strong background in theoretical computer science and motivation for studying questions in graph algorithms and computational complexity. Proficiency in English is required, while knowledge of French is not mandatory. For further information and details about the application process, interested candidates are strongly encouraged to contact the supervisors via email with informal inquiries.

References

[1] Rémy Belmonte, Eun Jung Kim, Michael Lampis, Valia Mitsou, and Yota Otachi. Grundy distinguishes treewidth from pathwidth. *SIAM J. Discret. Math.*, 36(3):1761–1787, 2022.

[2] Édouard Bonnet, Colin Geniet, Eun Jung Kim, Stéphan Thomassé, and Rémi Watrigant. Twin-width III: max independent set, min dominating set, and coloring. *SIAM J. Comput.*, 53(5):1602–1640, 2024.

[3] Marek Cygan, Fedor V. Fomin, Lukasz Kowalik, Daniel Lokshtanov, Dániel Marx, Marcin Pilipczuk, Michal Pilipczuk, and Saket Saurabh. *Parameterized Algorithms*. Springer, 2015.

[4] Ioannis Katsikarelis, Michael Lampis, and Vangelis Th. Paschos. Improved (in-)approximability bounds for d-scattered set. *J. Graph Algorithms Appl.*, 27(3):219–238, 2023.

[5] Michael Lampis. The primal pathwidth SETH. In Yossi Azar and Debmalya Panigrahi, editors, *Proceedings of the 2025 Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2025, New Orleans, LA, USA, January 12-15, 2025*, pages 1494–1564. SIAM, 2025.