PhD Proposal: Stable Matching with Incomplete Preferences

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The research line of this proposal is inspired by the French system used to match high school students to universities [7]. However, this topic applies to any centralized two-sided matching procedure where both sides of the market have preferences. Usually, these centralized procedures aim to select a matching that is stable, i.e., such that no student prefers another university over the one they are matched with, while that university would also be willing to accept the student. The literature on stable matching is extensive and includes contributions from both economics and computer science [5, 6]. The standard procedure used to assign students to universities is the well-known Gale-Shapley algorithm (`https://en.wikipedia.org/wiki/Gale-Shapley_algorithm`) [4]. In addition to ensuring stability, this procedure possesses several attractive properties. For instance, the matching it produces is optimal for students, meaning that no other stable matching is better for any student. Furthermore, this procedure is strategy-proof for students [3], meaning that no student can manipulate the outcome by misreporting their true preferences to obtain a better match (e.g., by strategically ranking a university higher because they have a greater chance of being accepted there). Strategy-proofness for students is a particularly desirable property, as it is closely related to fairness: it prevents well-informed students from gaining an advantage by manipulating the system, ensuring a more equitable process.

The desirable properties of the Gale-Shapley algorithm are the main reasons why this procedure is widely adopted for assigning students to universities. However, these properties rely on the assumption that the preferences of students over universities and those of universities over students are fully known. In other words, each student would need to provide a complete ranking of all universities. This task becomes impractical when the number of universities is very large, as it would require students to be fully informed about all university programs, sometimes across the entire country. For this reason, most systems based on the Gale-Shapley procedure ask students to rank only a limited set of universities, typically their top choices. However, this restricted version of Gale-Shapley does not preserve all the desirable properties described above. It is not strategy-proof, does not necessarily produce an optimal matching for students, and may even result in an unstable matching, as some students might

fail to rank certain universities simply because they are unaware of them before the application deadline.

In order to address these issues, we plan to study how similarity measures can help enrich revealed preferences. These similarity measures are well known in recommender systems [1] and can model common patterns in students' preferences. They can be derived from historical data on preferences already revealed by past students. Another approach, which may be used in combination with similarity measures, is to elicit students' unrevealed preferences, i.e., asking a limited number of targeted questions to assess their preferences over relevant universities [2].

References

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