Algorithmic Decision Theory within Discrete Complex Networks

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Abstract

Algorithmic decision theory becomes more and more important. We will present a theoretical and practical point of view to the role of networks and dynamic flow problems in complex environments. To support a decision support management a comfortable software implementation for solving multiobjective discrete control and dynamic flow problems on networks will be presented. As many processes from various economic areas such as information technology, transportation systems, multi-agent resource management, power distribution etc. can be modeled as such multiobjective discrete control and optimal flow problems on dynamic networks. For such kind of problems specific discrete algorithms as well as their robust implementations will be derived and analyzed. The talk will exploit a novel combination of discrete optimization methods and basic game-theoretic concepts to support an optimal decision support management process.

Keyword: Algorithmic Decision Theory, Network, Critical Infrastructure, Decision Support System

1 Introduction

Many decision processes from various economic areas such as information technology, transportation systems, multi-agent resource management, power distribution etc. can be modeled as multiobjective discrete control and optimal flow problems on dynamic networks. For such kind of problems efficient discrete algorithms as well as their robust implementations will be derived and analyzed. The talk will exploit a novel combination of discrete optimization methods and basic game-theoretic concepts. Furthermore dynamic programming techniques will be developed for such problems and two new algorithms for their solving will be derived and argued: Classification of necessary and sufficient conditions for the existence of solutions for the mentioned class of problem will be given. Polynomial and strongly polynomial algorithms for determining solutions will be elaborated. At the end we embed the algorithmic solutions to a decision support system.

2 Decision Support System

The aim of our paper is the analysis and the development of these advanced algorithms together with a comfortable software implementation for solving multiobjective discrete control and dynamic flow problems on certain networks. For that reason let us consider a simple time-discrete systems with a finite set of states and varying time of states transactions by a trajectory. We formulate a discrete optimal control problems with infinite time horizon for such systems. In a first step algorithms for finding the optimal stationary control and determining the optimal mean cost cycles in the graph of states transactions of dynamical systems are proposed. For that reason we study the following discrete optimal control problem with infinite time horizon and varying time of states transactions of a dynamical system:

Let the dynamical system L with the finite set of states $X \subseteq R^n$ be given, where at every discrete moment of time $t = 0, 1, 2, \dots$ the state of L is $x(t) \in X$. Assume, that the control of system L at each time-moment $t = 0, 1, 2, \dots$ for an arbitrary state $x(t)$ is made by using the vector of control parameters $u(t) \in R^m$ for which a feasible set $U_t\,(x(t))$ is given, i.e. $u(t) \in U_t\,(x(t))$. For arbitrary t and $x(t)$ on $U_t\,(x(t))$ it is defined an integer function

$$\tau : \; U_t\,(x(t)) \to N$$

which gives to each control $u(t) \in U_t\,(x(t))$ an integer value $\tau\,(u(t))$. This value expresses the time of system's passage from the state $x(t)$ to the state $x\big(t+\tau\,(u(t))\,\big)$ if the control $u(t) \in U_t\,(x(t))$ has been applied at the moment t for given state $x(t)$. The dynamics of the system L is described by the following system of difference equations

$$\begin{cases} t_{j+1} = t_j + \tau\,(u(t_j))\,; \\ x(t_{j+1}) = g_{t_j}\,(x(t_j), u(t_j))\,; \\ \quad u(t_j) \in U_{t_j}\,(x(t_j))\,; \\ \quad j = 0, 1, 2, \dots \end{cases} \tag{1}$$

where

$$x(t_0) = 0, \; t_0 = 0 \tag{2}$$

is a given starting representation of the dynamical system L. Here we suppose that the functions g_t and τ are known and t_{j+1} and $x(t_{j+1})$ are determined uniquely by $x(t_j)$ and $u(t_j)$ at every step j.

Let $u(t_j), \;\; j = 0, 1, 2, \dots,$ be a control, which generates the trajectory $x(0)$, $x(t_1), x(t_2), \dots, x(t_k), \dots$. For this control we define the mean integral-time cost by a trajectory

$$F_{x_0}\,(u(t)) = \lim_{k \to \infty} \frac{\sum\limits_{j=1}^{k-1} c_{t_j}\big(x(t_j), g_{t_j}\,(x(t_j), u(t_j))\,\big)}{\sum\limits_{j=0}^{k-1} \tau(u(t_j))} \tag{3}$$

where $c_{t_j}\big(x(t_j), g_{t_j}\,(x(t_j), u(t_j))\,\big) = c_{t_j}\,(x(t_j), x(t_{j+1}))$ represents the cost of system L to pass from the state $x(t_j)$ to the state $x(t_{j+1})$ at the stage $[j, j+1]$.

We consider the problem of finding the time-moments $t = 0, t_1, t_2, \ldots, t_{k-1}, \ldots$ and the vectors of control parameters $u(0), u(t_1), u(t_2), \ldots, u(t_{k-1}), \ldots$ which satisfy conditions (1), (2) and minimize functional (3).

In the case $\tau \equiv 1$ this problem became the control problem with unit time of states transactions from [1, 2, 5]. The problem of determining the stationary control with unit time of states transactions has been studied in [4, 5, 9]. In the mentioned papers it is assumed that $U_t(x(t))$, g_t and c_t do not depend on t, i.e. $g_t = g$, $c_t = c$ and $U_t(x) = U(x)$ for $t = 0, 1, 2, \ldots$. R. Bellman [1] showed that for the stationary case of the problem with unit time of states transactions there exists optimal stationary control $u^*(0)$, $u^*(1)$, $u^*(2)$, ..., $u^*(t)$, ..., such that

$$\lim_{k\to\infty} \frac{\sum\limits_{t=0}^{k-1} c\big(x(t), g\left(x(t), u^*(t)\right)\big)}{k} = \inf_{u(t)} \lim_{k\to\infty} \frac{\sum\limits_{t=0}^{k-1} c\big(x(t), g\left(x(t), u(t)\right)\big)}{k} = \lambda < \infty.$$

Furthermore in [5, 9] it is shown that the stationary case of the problem can be reduced to the problem of finding the optimal mean cost cycle in a graph of states transactions of dynamical system. Based on these results in [4, 5, 9] polynomial-time algorithms for finding optimal stationary control are proposed.

Here we develop the results mentioned above for the general case of the problem with arbitrary transit-time function τ. We show that this problem can by formulated as the problem of determining optimal mean cost cycles in the graph of states transactions of dynamical system for an arbitrary transition-time function on edges.

3 The Main Results - Decision Graph

The main results we propose are concerned with determining the stationary control in the general case for the problem from the first part. We show that this problem can be reduced to the following optimization problem on a so-called decision graph:

Let a strongly connected directed graph $G = (X, E)$ which represents the graph of states transactions of dynamical system L be given. An arbitrary vertex x of G corresponds to a state $x \in X$ and an arbitrary directed edge $e = (x, y) \in E$ expresses the possibility of system L to pass from the state $x(t)$ to the state $x(t + \tau(e))$, where $\tau(e)$ is the time of the system's passage from the state x to the state y through the edge $e = (x, y)$. So, on edge set E it is defined the function $\tau : E \to R^+$ which gives to each edge a positive number $\tau(e)$ which means that if the system L at the moment of time t has the state $x = x(t)$ then the system can reach the state y at the moment of time $t + \tau(e)$ if it passes through the edge $e = (x, y)$, i.e. $y = x(t + \tau(e))$. Additionally, on the edge set E it is defined the cost function $c : E \to R$, which gives to each edge the cost $c(e)$ of the system's passage from the state $x = x(t)$ to the state $y = x(t + \tau(e))$ for an arbitrary discrete moment of time t. So, finally we have that to each edge two numbers $c(e)$ and $\tau(e)$ are associated.

On G we consider the following problem:
To find a directed cycle C^* such that

$$\frac{\sum\limits_{e\in E(C^*)} c(e)}{\sum\limits_{e\in E(C^*)} \tau(e)} = \min_{\{C\}} \frac{\sum\limits_{e\in E(C)} c(e)}{\sum\limits_{e\in E(C)} \tau(e)}.$$

We show that this problem can be reduced to the following linear fractional problem:
To minimize

$$z = \frac{\sum\limits_{e\in E} c(e)\alpha(e)}{\sum\limits_{e\in E} \tau(e)\alpha(e)}$$

subject to

$$\sum_{e\in E^+(x)} \alpha(e) - \sum_{e\in E^-(x)} \alpha(e) = 0,\ x\in X;$$

$$\sum_{e\in E} \alpha(e) = 1;$$

$$\alpha(e) \geq 0,\ e\in E,$$

where $E^-(x) = \{e = (y,x) \in E | y \in X\}$; $E^+(x) = \{e = (x,y) | y \in E\}$.

Algorithms based on such approach for solving this problem are proposed. Additionally, a game theoretical approach for the considered problem is applied and some results from [7] are developed.

4 Perspectives: Uncertainty and Decision Support Systems

The design and optimization of comfortable decision support systems becomes more and more important. One disadvantage of many complex systems is that they often consist of a large amount of heterogeneous single applications that are inefficiently integrated into the overall process. This happens as such processes tend to grow over time, caused by an increase of complexity and supplementary demands by users for further functionalities, which leads to demands of new applications that are added to the system and need not always be compatible to the legacy applications. This results in process inefficiencies such as breakings in the media chain, high coordination effort, redundancy and an inefficient handling of information as the processing time increases. In case of threat on a critical infrastructure element, a fast and flexible acquisition, processing, and allocation of information are crucial. Flexibility, fast adaptability, and high process efficiency are central. Decision support management on complex networks structures -the basis of our mathemtical framework- is essential, even in the case of the protection of critical infrastructures.

5 Protection of Critical Infrastructures

Critical infrastructures are vital elements on which our daily live and society are based on, wherefore it is of great importance to pay a special attention to the protection of these elements. The following sectors can be identified as being critical infrastructure elements : Banking and Finance; Chemical Industry; Commercial Facilities; Commercial Nuclear Reactors, Materials, and Waste; Dams; Defence Industrial Base; Drinking Water and Wastewater Treatment Systems; Emergency Services; Energy; Food and Agriculture; Government Facilities; Information Technology; National Monuments and Icons; Postal and Shipping; Public Health and Healthcare; Telecommunications; and Transportation Systems. Break-downs or disturbances of such critical systems as a result of e.g. war, disaster, civil unrest, vandalism, or sabotage, may cause severe damage in the supply of a wide part of users linked to these systems and can have severe consequences to vital functions of the society. A definition is given in the "Patriot Act 2001 of the U.S.A" that describes critical infrastructures as :

"systems and assets, whether physical or virtual, so vital [] that the incapacity or destruction of such systems and assets would have a debilitation impact on security, national economic security, national public health or safety, or any combination of those matters."

Further definitions emphasize the interrelationship of the critical infrastructure elements :

"Critical infrastructures are the complex and highly interdependent systems, networks, and assets that provide the services essential in our daily life."

Our impression is that the mathematical framework presented here might be used to model such a decision process on a complex network.

6 Summary

We present a theoretical and practical point of view to the role of networks and dynamic flow problems in complex environments especially in the protection of critical infrastruvtures. We present the mathematical framework, the decision process on the network and characterizations of the specific algorithmic approach. With our approach we would like to support a decsion process on discrete networks which might also support a better crisis management. First results would be presented and discussed.

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