Decision Policy Design as Pareto-Minimization of Infeasible Lower Bounds

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A Configuration Problem

- Catalogs for components a and b

y_a	$y_{1,a}$	$y_{2,a}$	$y_{3,a}$
a_1	10	20	30
a_2	20	30	10

y_b	$y_{1,b}$	$y_{2,b}$	$y_{3,b}$
b_1	10	10	30
b_2	20	30	10

- Configure a system out of a and b s.t.
  - the sum of the $y_{1,i}$'s is greater than p_1
  - the sum of the $y_{2,i}$'s is greater than p_2
  - the sum x of the $y_{3,i}$'s is maximized
- Interactive usage:
  - the parameters p_1 and p_2 are the user input
  - the optimum x is the user output

Configuration Rules

- Rules for making optimal decisions:

  1. **if** $p_1 \geq 0 \wedge p_1 \leq 20 \wedge p_2 \geq 0 \wedge p_2 \leq 30$ **then** $x := 60$.
  2. **if** $p_1 \leq 30 \wedge p_2 \geq 31 \wedge p_2 \leq 50$ **then** $x := 40$.
  3. **if** $p_1 \leq 40 \wedge p_2 \geq 51 \wedge p_2 \leq 60$ **then** $x := 20$.
  4. **if** $p_1 \geq 0 \wedge p_2 \geq 61$ **then** $x := 0$.
  5. **if** $p_1 \geq 21 \wedge p_1 \leq 30 \wedge p_2 \leq 50$ **then** $x := 40$.
  6. **if** $p_1 \geq 31 \wedge p_1 \leq 40 \wedge p_2 \leq 60$ **then** $x := 20$.
  7. **if** $p_1 \geq 41 \wedge p_2 \geq 0$ **then** $x := 0$.

- Interactive usage:
  - fast optimal online decision making
  - without solving optimization problems online

Changing Catalogs

- New component types are added

y_a	$y_{1,a}$	$y_{2,a}$	$y_{3,a}$
a_1	10	20	30
a_2	20	30	10
a_3	20	20	10

y_b	$y_{1,b}$	$y_{2,b}$	$y_{3,b}$
b_1	10	10	30
b_2	20	30	10
b_3	10	30	20

- Impact of the change
  - are the rules still making feasible decisions?
  - are the rules still making optimal decisions?
  - if no, how to change the rules?

Outline

- Decision policies
- Design of policies from models
  1. Policy design by exhaustive optimization
  2. Policy design by Pareto-optimization
- Computing the policy by a dual approach

Decision under conditions

- Which are the possible decisions?
  - described by a decision space $\mathcal{X}$
  - *e.g., choose a category among Gold, Silver, Platinum*
- Which parameter may influence the choice?
  - described by a parameter space $\mathcal{P}$
  - *e.g., the cart value of the customer*
- When is which decision feasible?
  - described by a subset $X(p)$ of $\mathcal{X}$ for each $p \in \mathcal{P}$
  - *e.g., Silver is always possible*
    Platinum is possible if the cart value is at least 1000
    Gold is possible if the cart value is at least 500

Decision policy

- What is a decision policy?
  - chooses a single feasible decision for each parameter value
  - a function $\pi : \mathcal{P} \rightarrow \mathcal{X}$ s.t. $\pi(p) \in X(p)$ for all $p \in \mathcal{P}$
  - can adequately be represented by rules
- Example
  - $\textbf{if } value \geq 0 \wedge value < 800 \textbf{ then } category = Silver.$
  - $\textbf{if } value \geq 800 \wedge value < 1000 \textbf{ then } category = Gold.$
  - $\textbf{if } value \geq 1000 \wedge value \leq 2000 \textbf{ then } category = Platinum.$
  - $\textbf{if } value \geq 2000 \textbf{ then } category = Gold.$

Rational decision policy

- Preferences between decisions
  - total preference order $\succsim^x$ on the decision space
  - *e.g., Platinum is preferred to Gold, which is preferred to Silver*
- What is a rational policy?
  - a rational policy chooses an optimal decision from $X(p)$ for each p
  - *i.e., there is no* $x^* \in X(p)$ **s.t.** $x^* \succ^x \pi(p)$
- Example
  - $\mathbf{if}\ value \geq 0 \wedge value < 800\ \mathbf{then}\ category = Silver.$
  - $\mathbf{if}\ value \geq 800 \wedge value < 1000\ \mathbf{then}\ category = Gold.$
  - $\mathbf{if}\ value \geq 1000\ \mathbf{then}\ category = Platinum.$

Dominance between parameters

- Strictness order
  - partial order $\succsim^p$ on the parameter space
  - $p_1 \succsim^p p_2$ implies $X(p_1) \subseteq X(p_2)$
- Examples
  - less budget means less options
  - smaller cart value will reduce the feasible categories
  - less votes mean less seats
- Simplification of rules
  - $\textbf{if } value \geq 0 \textbf{ then } category \succsim^x Silver.$
  - $\textbf{if } value \geq 800 \textbf{ then } category \succsim^x Gold.$
  - $\textbf{if } value \geq 1000 \textbf{ then } category \succsim^x Platinum.$

cf. [Greco, Matarazzo, Slowinsky, EJOR01]

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Acquisition of policies

1. from experts
   - domain is well-understood by experts
   - use rule authoring or knowledge acquisition
2. from data:
   - domain is not well-understood
   - use data analysis, data mining, rule discovery
3. from models:
   - domain deals with artifacts designed by humans
   - example: product confi guration, complex pricing
   - use multi-criteria optimization

Configuration Model

1. Variables y
   - variables for parameters $y_{\mathbf{p}}$
   - variables for decision $y_{\mathbf{x}}$
   - auxiliary variables, e.g. $y_{i,j}$
2. Constraints C
   - $(y_i, y_{1,i}, y_{2,i}, y_{3,i})$ is a line in catalog i
   - $y_{1,a} + y_{1,b} \geq y_{\mathbf{p}_1}$
   - $y_{2,a} + y_{2,b} \geq y_{\mathbf{p}_2}$
   - $y_{3,a} + y_{3,b} = y_{\mathbf{x}}$
3. Objective
   - maximize $y_{\mathbf{x}}$

Policy Design

- Solutions under p:

  set of solutions y of C that satisfy $y_{\mathbf{p}} = p$

- Feasible decisions:

  $x \in X(p)$ iff

  1. x is supported by a solution y of C under p, i.e. $y_{\mathbf{x}} = x$
  2. or x is the worst decision $x^{\perp}$ (which is unsupported).

- Optimal decisions

  x^* is an optimal decision in $X(p)$ iff there is no solution y under p s.t. $y_{\mathbf{x}} \succ^x x^*$

classic combinatorial optimization problem!

Exhaustive Optimization

- Approach
  - for each $p \in \mathcal{P}$, find the optimal decision x^* in $X(p)$
  - we know that all solutions of C satisfy:

    $$y_{\mathbf{p}} = p \Rightarrow y_{\mathbf{x}} \precsim^{x} x^*$$
  - we can represent this implication by a rule

    $$\textbf{if } y_{\mathbf{p}} = p \textbf{ then } y_{\mathbf{x}} := x^*.$$
- Cost
  - the number of optimization problem is equal to the size of $\mathcal{P}$
  - the number of rules is equal to the size of $\mathcal{P}$
  - $\mathcal{P}$ may have exponential size

Example

Many rules are similar:

- $y_{\mathbf{p}_1} = 0 \wedge y_{\mathbf{p}_2} = 0 \Rightarrow y_{\mathbf{x}} \precsim^x 60$
- $y_{\mathbf{p}_1} = 0 \wedge y_{\mathbf{p}_2} = 1 \Rightarrow y_{\mathbf{x}} \precsim^x 60$
- $\ldots$
- $y_{\mathbf{p}_1} = 0 \wedge y_{\mathbf{p}_2} = 30 \Rightarrow y_{\mathbf{x}} \precsim^x 60$
- $y_{\mathbf{p}_1} = 0 \wedge y_{\mathbf{p}_2} = 31 \Rightarrow y_{\mathbf{x}} \precsim^x 50$
- $y_{\mathbf{p}_1} = 0 \wedge y_{\mathbf{p}_2} = 32 \Rightarrow y_{\mathbf{x}} \precsim^x 50$
- $\ldots$
- $y_{\mathbf{p}_1} = 0 \wedge y_{\mathbf{p}_2} = 50 \Rightarrow y_{\mathbf{x}} \precsim^x 50$
- $y_{\mathbf{p}_1} = 0 \wedge y_{\mathbf{p}_2} = 51 \Rightarrow y_{\mathbf{x}} \precsim^x 30$

Relax conditions

- Original Rules
  - $y_{\mathbf{p}} = p \Rightarrow y_{\mathbf{x}} \precsim^{x} x^*$
- Relaxing condition:
  - $y_{\mathbf{p}} \succsim^{p} p \Rightarrow y_{\mathbf{x}} \precsim^{x} x^*$
- Strict upper bound action:
  - $y_{\mathbf{p}} \succsim^{p} p \Rightarrow y_{\mathbf{x}} \prec^{x} u^*$
    where u^* is the worst element in $\{x \in X(p) \mid x \succ^{x} x^*\}$
- Rule:
  - $\mathbf{if}\ y_{\mathbf{p}} \succsim^{p} p\ \mathbf{then}\ y_{\mathbf{x}} := \max(y_{\mathbf{x}}, x^*)$.

Dominance Phenomenon

- Two rules
  - $y_{\mathbf{p}} \succsim^p p^{(1)} \Rightarrow y_{\mathbf{x}} \prec^x u^{(1)}$
  - $y_{\mathbf{p}} \succsim^p p^{(2)} \Rightarrow y_{\mathbf{x}} \prec^x u^{(2)}$
- Dominance
  - suppose $p^{(1)} \succsim^p p^{(2)}$ and $u^{(1)} \succsim^x u^{(2)}$
  - then the second rule makes the fi rst rule redundant
  - if a rule r_1 makes a rule r_2 redundant, but not vice versa, then we can delete r_2

The Policy Design Problem

- Pareto-dominance
  - combined space $\mathcal{Z} := \mathcal{P} \times \mathcal{X}$
  - $(p^{(1)}, u^{(1)}) \succsim^z (p^{(2)}, u^{(2)})$ iff $p^{(1)} \succsim^p p^{(2)}$ and $u^{(1)} \succsim^x u^{(2)}$
- Infeasible lower bounds
  - (p, u) from $\mathcal{Z}$ is a candidate iff
  - all solutions y of C satisfy $y_{\mathbf{p}} \succsim^p p \Rightarrow y_{\mathbf{x}} \prec^x u$ iff
  - no solution y of C satisfi es $y_{\mathbf{p}} \succsim^p p \wedge y_{\mathbf{x}} \succsim^x u$ iff
  - no solution y of C satisfi es $y_{\mathbf{px}} \succsim^z (p, u)$
- Pareto-minimal infeasible lower bounds
  - fi nd all infeasible lower bounds (p, u) that are Pareto-minimal w.r.t. $\succ^z$

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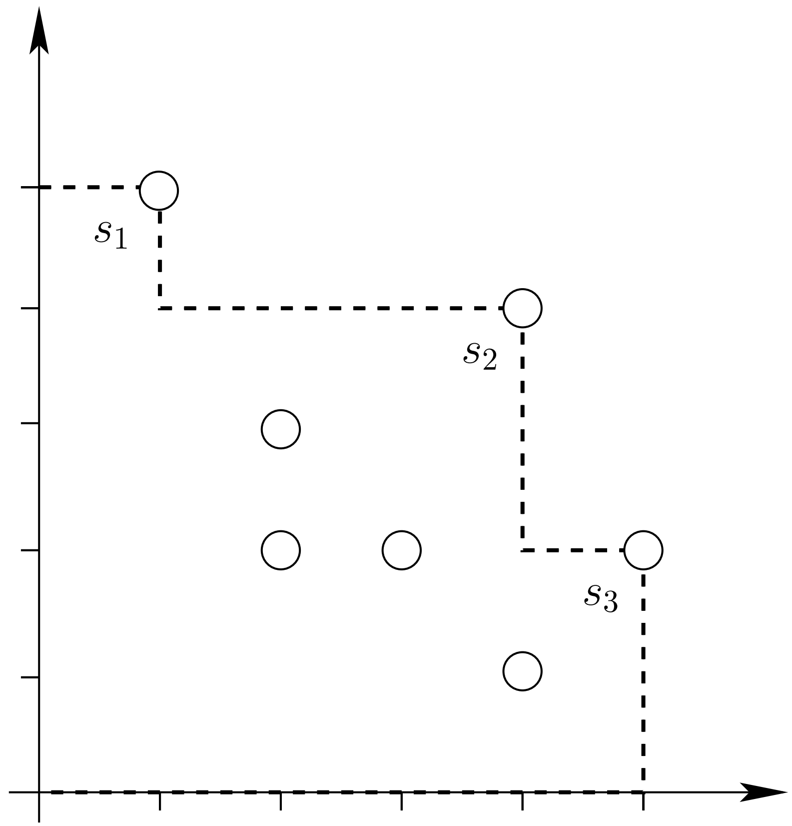
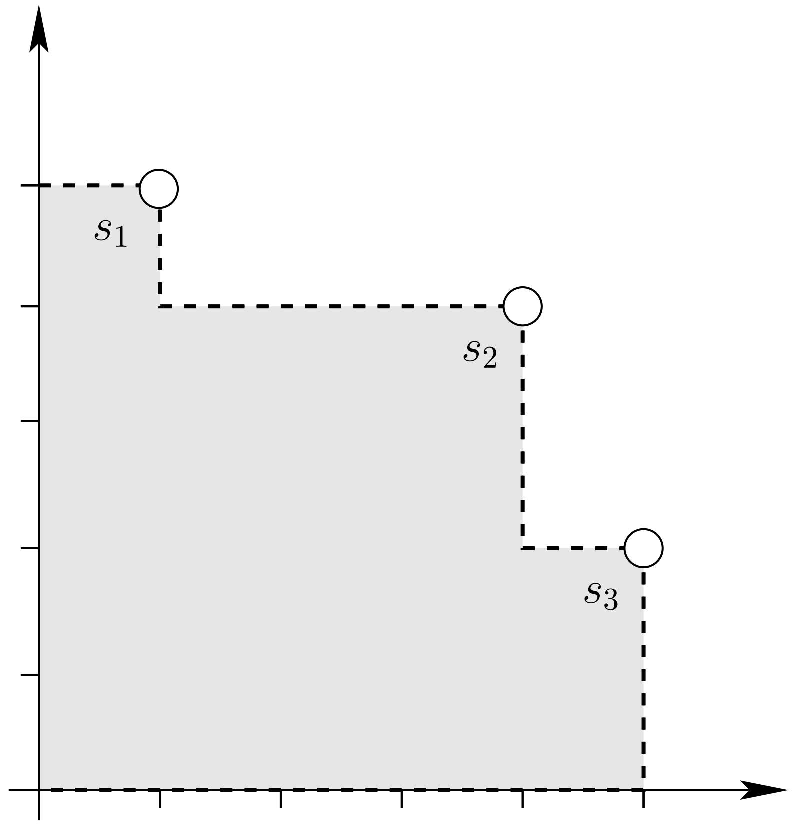
Dual Approach: Step I



Pareto-maximal solutions:

find all Pareto-maximal elements of $\mathcal{Z}$ that are supported by a solution

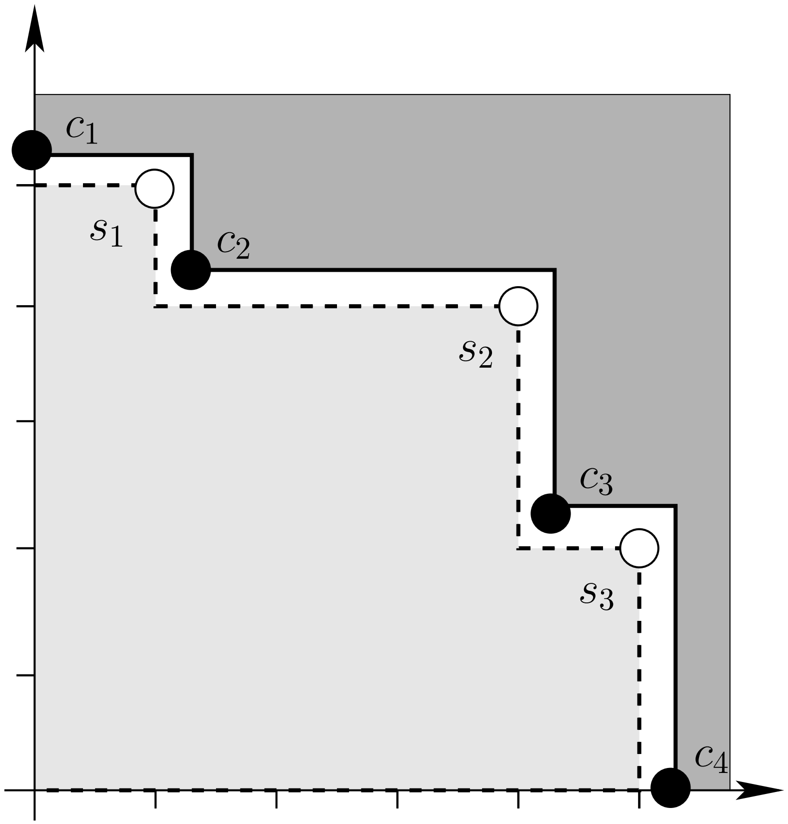
Dual Approach: Step II



Dominated region ("feasible lower bounds"):

part of $\mathcal{Z}$ that is weakly dominated by some Pareto-maximal solution

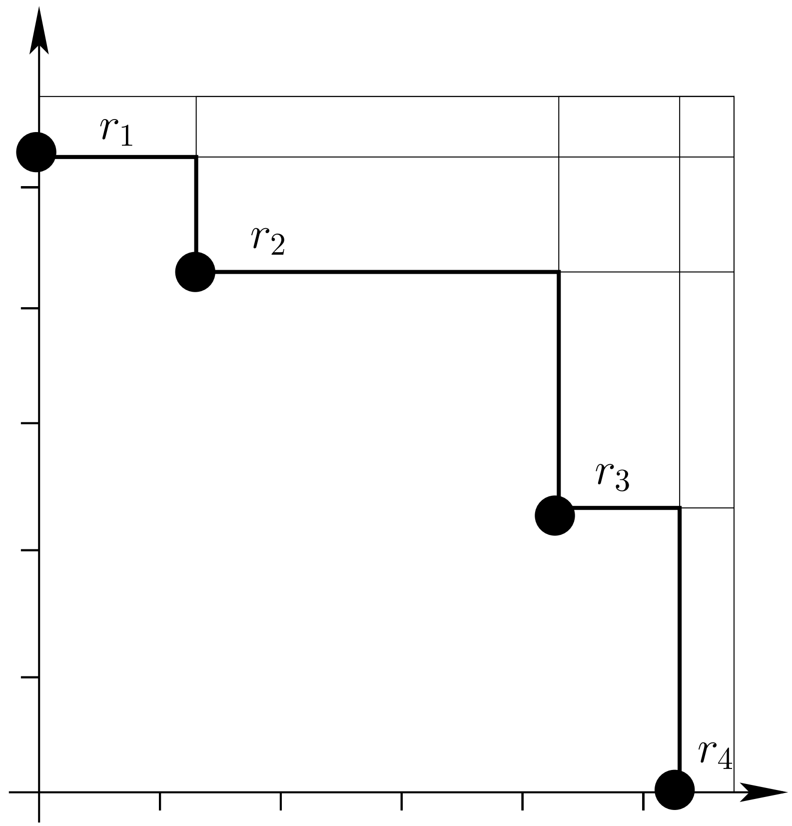
Dual Approach: Step III



Nondominated region (“infeasible lower bounds”)

part of $\mathcal{Z}$ that is not weakly dominated by some Pareto-maximal solution

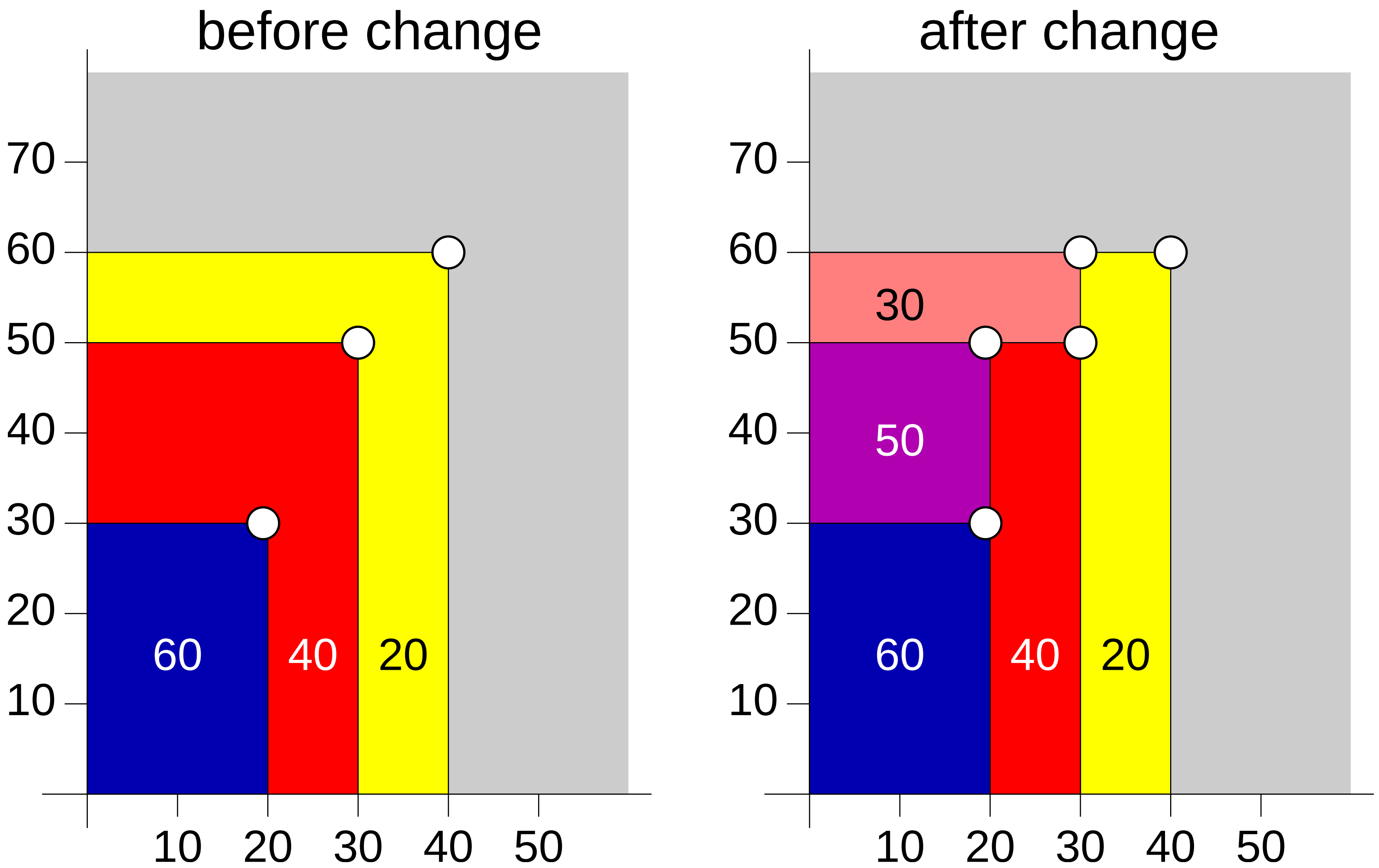
Dual Approach: Step IIII



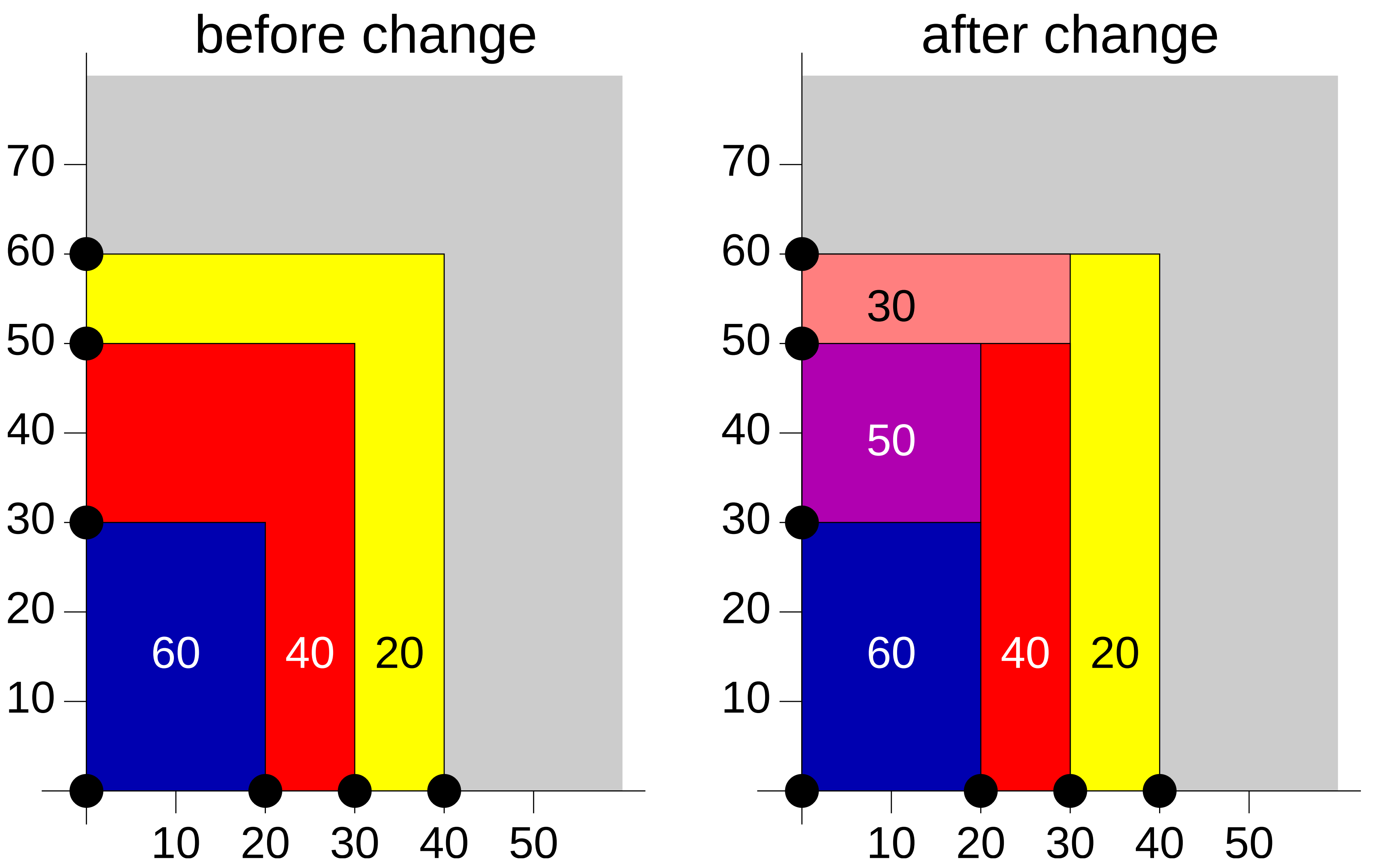
Dual frontier:

Pareto-minimal elements of the non-dominated region

Pareto-frontier



Dual frontier



Rules before catalog change

1. **if** $p_1 \geq 0 \wedge p_2 \geq 0$ **then** $x := min(x, 60)$.
2. **if** $p_1 \geq 0 \wedge p_2 \geq 31$ **then** $x := min(x, 40)$.
3. **if** $p_1 \geq 0 \wedge p_2 \geq 51$ **then** $x := min(x, 20)$.
4. **if** $p_1 \geq 0 \wedge p_2 \geq 61$ **then** $x := min(x, 0)$.
5. **if** $p_1 \geq 21 \wedge p_2 \geq 0$ **then** $x := min(x, 40)$.
6. **if** $p_1 \geq 31 \wedge p_2 \geq 0$ **then** $x := min(x, 20)$.
7. **if** $p_1 \geq 41 \wedge p_2 \geq 0$ **then** $x := min(x, 0)$.

Rules after catalog change

1. **if** $p_1 \geq 0 \land p_2 \geq 0$ **then** $x := min(x, 60)$.
2. **if** $p_1 \geq 0 \land p_2 \geq 31$ **then** $x := min(x, 50)$.
3. **if** $p_1 \geq 0 \land p_2 \geq 51$ **then** $x := min(x, 30)$.
4. **if** $p_1 \geq 0 \land p_2 \geq 61$ **then** $x := min(x, 0)$.
5. **if** $p_1 \geq 21 \land p_2 \geq 0$ **then** $x := min(x, 40)$.
6. **if** $p_1 \geq 31 \land p_2 \geq 0$ **then** $x := min(x, 20)$.
7. **if** $p_1 \geq 41 \land p_2 \geq 0$ **then** $x := min(x, 0)$.

Conclusion

- New method for policy acquisition
  - rules are derived from a domain model
  - by Pareto-minimization of ‘infeasible lower bounds’
- Dual approach
  - determine Pareto-maximal solutions
  - use them to defi ne the constraints of the dual problem
  - determine Pareto-minimal solutuions of the dual problem
- Applications
  - web-based confi guration
  - automated pricing