From Decision Theory to Combinatorial Optimization: Problems and Algorithms in Graphs

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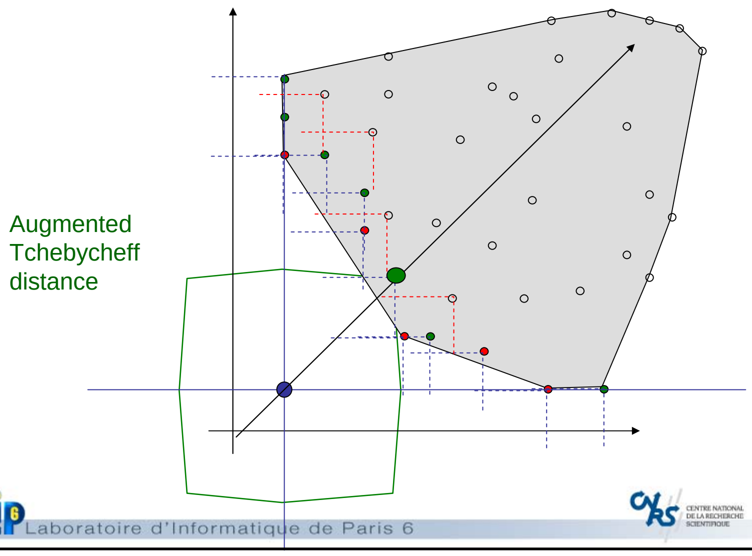
Outline

1. Examples and motivation
2. Using decision models in multiobjective graphs problems
   - Lorenz-optimal paths
   - OWA-optimal assignment/transportation
   - Choquet-optimal spanning trees
3. Approximation of preferred solutions
4. Conclusion

1. Examples and motivations

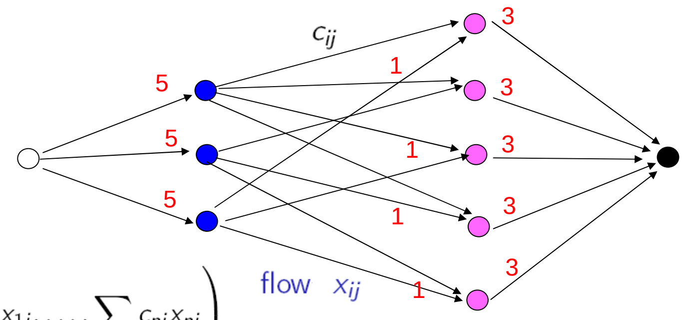
- Compromise search in multiobjective optimization
- Equity in multiagent assignment problems
- Robustness in optimization under uncertainty

Compromise search in multiobjective (combinatorial) optimization



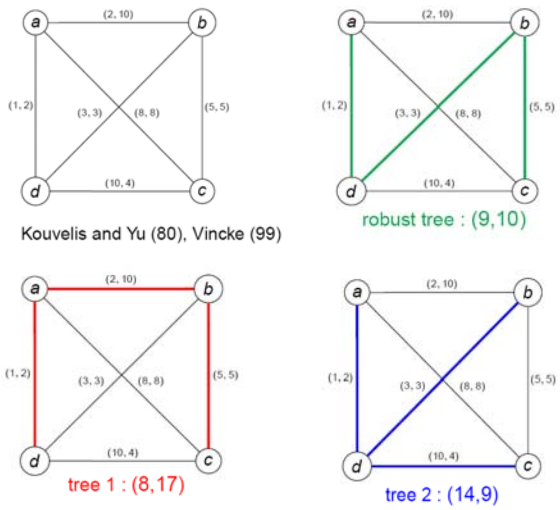
Fairness in multiagent assignment/transportation problems

- Paper assignment problems [e.g., Goldsmith and Sloan 07, Wang et al.08]
- Allocation of indivisible goods [e.g. Bouveret and Lang, 05]
- Matching in social networks (e.g. Meetic)



$$\left(\sum_j c_{1j}x_{1j}, \ldots, \sum_j c_{nj}x_{nj}\right)$$

Robustness in optimization under uncertainty



Kouvelis and Yu (80), Vincke (99)

Multiobjective combinatorial optimization

- combinatorial structure of the set of alternatives
- multidimensional evaluation of solutions $(x_1, \ldots, x_n)$
  - *multicriteria analysis :* x_i performance w.r.t criterion i
  - *multiagent decision-making :* x_i satisfaction of agent i
  - *decision under uncertainty :* x_i consequence in scenario s_i

Tools

- preference models and optimality concepts
- algorithms to find preferred solutions on combinatorial domains

$\rightarrow$ Algorithmic Decision Theory

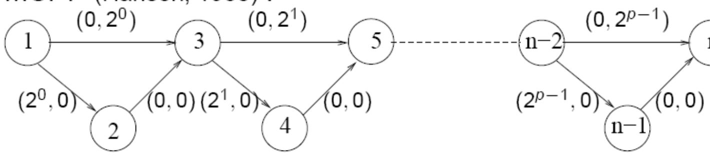
Some references in MOCO

Main topic : determination of the set of Pareto Optimal Solutions

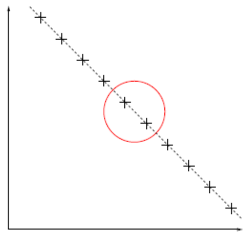
- *shortest paths :* Vincke (75), Hansen (80), Martins (84), Warburton (87), Stewart and White (91), Gandibleux et al. (06)
- *spanning trees :* Corley (83), Serafini (86), Hamacher and Ruhe (94), Andersen (96), Ramos and al. (98), Zhou and Gen (99), Knowles and Corne (01), Steiner and Radzik (06), Sourd and Spanjaard (08)
- *knapsack :* Ulungu and Teghem (97), Climaco, Figueira and Martins (01), Captivo et al. (03), Kumar and Barnajee (06), Bazgan, Hugot and Vanderpooten (08), Perny and Spanjaard (08)
- *assignment :* R Malhotra et al. (82), D. Tuyttens, J. Teghem and Ph. Fortemps (00)
- *scheduling :* Tkindt (02)
- Books, surveys : Ehrgott (00), Ehrgott and Gandibleux (00,04)

Pareto-optimal paths: an intractable problem

MSPP (Hansen, 1980) :



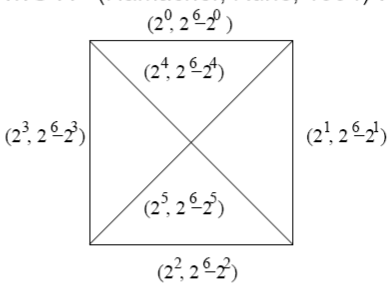
$$\forall P,\ c_1(P) + c_2(P) = \sum_{i=0}^{p-1} 2^i$$

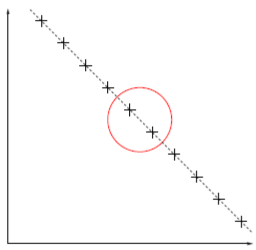


The number of Pareto-optimal solutions exponentially grows with the size of the graph (number of nodes)

Pareto-optimal spanning trees: an intractable problem

MSTP (Hamacher, Ruhe, 1994) :





$$\forall T,\ c_1(T) + c_2(T) = (n-1)2^m$$

The number of Pareto-optimal solutions exponentially grows with the size of the graph (number of nodes)

Exploration of Pareto-optimal solutions

Different approaches :

1. Complete generation :
   - might be intractable in some cases
   - might be useless (many Pareto solutions are not satisfactory)
2. Decision theoretic approach : use a more sophisticated and discriminating model
   - directly focuses on the region of interest for the DM
   - requires a significant work in elicitating the model
3. Progressive exploration :
   Interactive approach (Zionts and Wallenius 76, Steuer, 83, 86, Vanderpooten and Vincke 89)
   - avoids complete enumeration, directly focuses on the region of interest at the current step
   - solutions might evolve with preferences during the search

Preference models for vector optimization

DOMINANCE RELATIONS

- Pareto dominance (MCDM/Social Choice)
- ε−dominance (MOCO)
- Lorenz dominance (Equity measurement)
- Second Order Stochastic Dominance (Risk-aversion)

SCALARIZING FUNCTIONS

- Weighted sum
- Additive utility (MAUT), EU (risk)
- Max, Leximax (Bottleneck, Robust optimization)
- Tchebycheff norm (MCDM)
- OWA, WOWA (Fairness, MCDM)
- Yaari's model, RDU (Risk)
- Choquet integral (Uncertainty, MCDM)

2. Using decision models in multiobjective combinatorial optimization: a research program

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths										
Trees										
Flows										
Knapsack										
....										

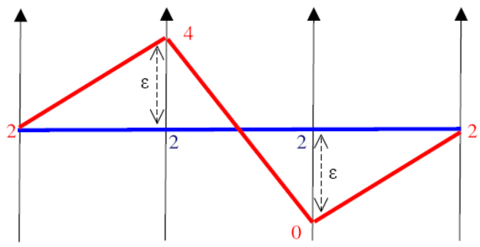
2.1 Lorenz-optimal paths

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths			1							
Trees										
Assign										
Knapsack										
....										

Aim: favouring well-balanced cost distributions

DEFINITION (TRANSFER PRINCIPLE)

Let $x \in \mathbb{R}^m_+$ s.t. $x_i > x_j$ for some $i, j \in \{1, \ldots, m\}$:
for all ε such that $0 < \varepsilon < x_i - x_j$: $x - \varepsilon e_i + \varepsilon e_j \succ x$



Generalized Lorenz dominance

DEFINITION (L-DOMINANCE)

$\forall x, y \in \mathbb{R}^m_+,\ x \succeq_L y \iff L(x) \succeq_P L(y)$

where $L(x) = (x_{(1)}, x_{(1)} + x_{(2)}, \ldots, x_{(1)} + x_{(2)} + \ldots + x_{(m)})$
with $x_{(1)} \geq x_{(2)} \geq \ldots \geq x_{(m)}$

(11, 9, 10) $>_L$ (6, 10, 15) because (11, 21, 30) $>_P$ (15, 25, 31)

THEOREM (HARDY, LITTLEWOOD AND POLYA, 1929, CHONG, 1976)

For all $x, y \in \mathbb{R}^m_+$, if $x \succ_P y$, or if x obtains from y by an admissible transfer then $x \succ_L y$.

Conversely, if $x \succ_L y$, there exists a sequence of admissible transfers and/or Pareto improvements to transform y into x.

- Lorenz dominance refines Pareto dominance
- Favours well-balanced solutions (transfer principle)

L-optimality: complexity issues

see Perny, Spanjaard and Storme (AOR, 06)

PROPOSITION

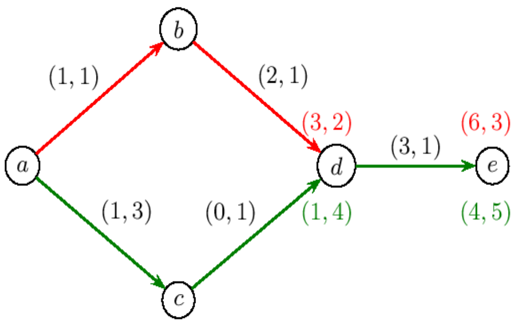
The problem of finding L-efficient paths in a graph is, in worst case, intractable, i.e. it requires a number of operations which grows exponentially with the size of the instance.

PROPOSITION

Deciding whether there exists a path whose cost distribution L-dominates a given cost-vector is an NP-complete decision problem.

The same results hold for spanning tree

L-dominance and the Bellman principle



(3, 5) (4, 5)

$(3,2) \succ_L (1,4)$

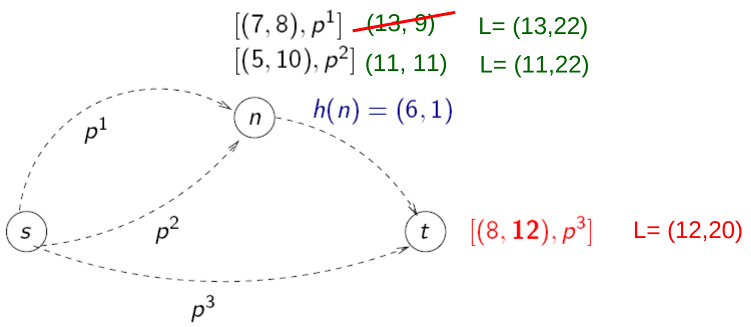
(5, 9) (6, 9)

$(4,5) \succ_L (6,3)$

A simple label-setting algorithm

Focused multiobjective search (paths)

- A Multiobjective version of Dijkstra Algorithm [Martins'84]
- prune subpaths L-dominated by an already detected solution path.



Numerical tests for L-optimal paths

(random instances, graph density ~ 50%)

m	#nodes	# L-opt	time (s)
2	1000	2.20	0.12
	3500	2.25	1.75
	6000	2.45	5.75
5	1000	5.10	0.25
	3500	5.70	4.14
	6000	6.60	13.69
10	1000	10.75	0.55
	3500	14.15	9.47
	6000	13.5	30.97

Refining Lorenz dominance

Neutrality. For all x, y in X, $L(x) = L(y) \Rightarrow x \sim y$

L-Monotoncity. For all x, y in X,$L(x) \succ_P L(y) \Rightarrow x \succ y$

Complete weak-order. $\succsim$ is reflexive, transitive and complete.

Continuity. Let $L, M, N \in L(X)$ such that $L \succ M \succ N$. There exists $\alpha, \beta \in]0,1[$ such that :

$$\alpha L + (1-\alpha)N \succ M \succ \beta L + (1-\beta)N$$

Independence. Let L, M, N belong to $L(X)$. Then, for all $\alpha \in]0,1[$:

$$L \succ M \Longrightarrow \alpha L + (1-\alpha)N \succ \alpha M + (1-\alpha)N$$

OWA as a measure of inequality

Theorem (Perny, Spanjaard, Storme 06)

$\succsim$ satisfies Neutrality, L-monotonicity, complete weak-order, continuity and independence iff it is reprensetable by an OWA function with strictly decreasing weights $w_i > w_{i+1}$ for all i.

$$OWA(x_1, \ldots, x_n) = \sum_{i=1}^{n} w_i x_{(i)}$$

with $x_{(1)} \geq x_{(2)} \geq \ldots \geq x_{(n)}$

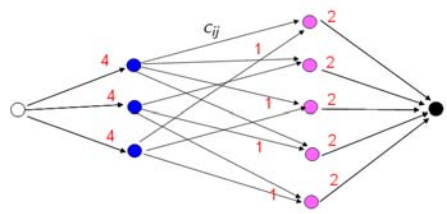
related results :

- max OWA with decreasing weights implies L-optimality, Ogryczak(2000) on equitable location problems
- characterization of Gini indices (measuring income inequality) obtained by Weymark(1981)
- ...

2. OWA-optimal assignment/transportation

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths			1							
Trees										
Assign							2			
Knapsack										
....										

Fair assignment problems



$$\text{Min} \quad \text{OWA}\left(\sum_j c_{1j}x_{1j}, \ldots, \sum_j c_{nj}x_{nj}\right)$$

$$\sum_{j=1}^{n} x_{ij} \le m \quad i = 1 \ldots n$$
$$\sum_{i=1}^{n} x_{ij} \le p \quad j = 1 \ldots n$$
$$x_{ij} \in \{0,1\} \quad i = 1 \ldots n,\ j = 1 \ldots n$$

An example: WS vs OWA in multiagent assignment problems

$$\begin{pmatrix} 5 & 8 & (4) & 9 & 7 \\ 1 & (3) & 2 & 7 & 8 \\ (3) & 9 & 2 & 9 & 5 \\ 10 & 1 & 3 & (3) & 4 \\ 5 & 1 & 7 & 7 & (3) \end{pmatrix}$$

WS-opt $7, 1, 2, 3, 1$ de coût WS = 14/5

OWA-opt $(4), (3), (3), (3), (3)$ WS = 16/5

LP formulation of OWA-optimization

$y = (y_1, \dots, y_n) \qquad y_{(1)} \geq y_{(2)} \dots \geq y_{(n)}$

$$\text{OWA}(y) = \sum_{k=1}^{n} w_i y_{(k)} = \sum_{k=1}^{n} w'_k L_k(y) \qquad w' = (w_1 - w_2, \dots, w_{n-1} - w_n, w_n) > 0$$

$L_k(y) =$

$$\begin{array}{ll} \text{Max } \sum_{i=1}^{n} \alpha_i^k y_i & \text{Min } kr_k + \sum_{i=1}^{n} b_i^k \\ \sum_{i=1}^{n} \alpha_i^k = k & r_k + b_i^k \geq y_i \\ 0 \leq \alpha_i^k \leq 1 \quad i = 1 \dots n & b_i^k \geq 0 \quad i = 1 \dots n \end{array}$$

dual

$$\text{Min } \sum_{k=1}^{p} w'_k \left(kr_k + \sum_{i=1}^{n} b_i^k \right)$$

(Ogryczak, 07)

$r_k + b_i^k \geq y_i$

$b_i^k \geq 0$

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A mixed-integer LP formulation of the OWA-optimal assignment problem

$$\text{Min} \sum_{k=1}^{n} w'_k \left(k r_k + \sum_{i=1}^{n} b_i^k \right)$$

$\sum_{j=1}^{n} x_{ij} \leq \text{m}$	$i = 1 \ldots n$
$\sum_{i=1}^{n} x_{ij} \leq \text{p}$	$j = 1 \ldots n$
$r_k + b_j^k \geq \sum_{i=1}^{n} c_{ij} x_{ij}$	$j = 1 \ldots n,\ k = 1 \ldots n$
$b_j^k \geq 0$	$j = 1 \ldots n,\ k = 1 \ldots n$
$x_{ij} \in \{0, 1\}$	$i = 1 \ldots n,\ j = 1 \ldots n$

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Numerical tests with Cplex for OWA assignment

n =	100	200	300	400	500	600	700	800	900
t (OWA)	.98	2.37	10.6	23.0	32.4	57.7	84.5	158	227

Times (in seconds) for fair assignment problems with n agents, costs in {1, …, 20}

n =	100	200	300	400	500	600	700	800	900	1000	1100
t	.23	1.58	4.8	10	20	37	57	93	151	222	361

Times (in seconds) for paper assignment problems with n reviewers, 3n papers

costs in {1, …, 5}, matrix density 20%, max nb of paper per agent = 5.

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2.3 Choquet-optimal spanning trees

[Galand, Perny, Spanjaard, 08]

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths			1							
Trees										3
Assign							2			
Knapsack										
....										

The Choquet Expected Disutility model

Capacity

set function $v : 2^N \rightarrow [0,1]$ such that :

- $v(\emptyset) = 0,\ v(N) = 1$
- $\forall A, B \in 2^N$ such that $A \subseteq B,\ v(A) \leq v(B)$

Choquet Integral

$$C_v(x) = \sum_{i=1}^{n} \left[v(X_{(i)}) - v(X_{(i+1)}) \right] x_{(i)} = \sum_{i=1}^{n} \left[x_{(i)} - x_{(i-1)} \right] v(X_{(i)})$$

where $X_{(i)} = \{j \in S,\ x_j \geq x_{(i)}\} = \{(i),(i+1),\ldots,(n)\}$

Choquet Expected Disutility $\psi_v^w(x) = C_v(w(x_1), \ldots, w(x_n))$

CED includes multiple models as special cases

$$\psi_v^w(x) = \sum_{i=1}^{n} [v(X_{(i)}) - v(X_{(i+1)}]\, w(x_{(i)}) \quad \forall i, x_{(i)} \leq x_{(i+1)} \qquad X_{(i)} = \{(i), (i+1), \ldots, (n)\}$$

- Expected disutility (vNM, 47) : additive capacity $v(X) = \sum_{k \in X} v_k$
  $\psi_v^w(x) = \sum_{i=1}^n v_i w(x_i)$
- OWA (Yager, 88) : symmetric capacity $v(X) = \varphi(|X|)$ and $w(x) = x$
  $\psi_v^w(x) = \sum_{i=1}^n w_i x_{(i)}$ with $w_i = \varphi(n-i+1) - \varphi(n-i)$
- Yaari's model (Yaari, 87) or WOWA (Torra,97) : capacity $v(X) = \varphi(\sum_{i \in X} v_i)$ and $w(x) = x$
  $\psi_v^w(x) = \sum_{i=1}^n \left[\varphi(\sum_{k \in X_{(i)}} v_k) - \varphi(\sum_{k \in X_{(i+1)}} v_k)\right] x_{(i)}$
- RDU (Quiggin, 89) : capacity $v(X) = \varphi(\sum_{i \in X} v_i)$
  $\psi_v^w(x) = \sum_{i=1}^n \left[\varphi(\sum_{k \in X_{(i)}} v_k) - \varphi(\sum_{k \in X_{(i+1)}} v_k)\right] w(x_{(i)})$

Compromise search, fairness or uncertainty aversion

Preference for interior points

$\forall x^1, \ldots, x^n \in \mathbb{N}^n$, $\forall \alpha_1, \ldots, \alpha_n \geq 0$ s.t. $\sum_{i=1}^n \alpha_i = 1$,

$$[x^1 \sim x^2 \sim \ldots \sim x^n] \Rightarrow \sum_{i=1}^{n} \alpha_i x^i \succsim x^k, \quad k = 1, \ldots, n$$

$$(30, 30, 0) \sim (30, 0, 30) \sim (0, 30, 30)$$

$$(20, 20, 20) = 1/3(30, 30, 0) + 1/3(30, 0, 30) + 1/3(0, 30, 30)$$

should be preferred

Compromise search, fairness or uncertainty aversion

Preference for interior points

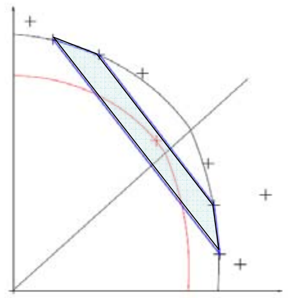
$\forall x^1, \dots, x^n \in \mathbb{N}^n, \;\; \forall \alpha_1, \dots, \alpha_n \geq 0$ s.t. $\sum_{i=1}^{n} \alpha_i = 1$,

$$[x^1 \sim x^2 \sim \dots \sim x^n] \Rightarrow \sum_{i=1}^{n} \alpha_i x^i \succsim x^k, \quad k = 1, \dots, n$$

EXAMPLE :

With $v(\{1\}) = 0.8$, $v(\{2\}) = 0.4$ and $v(\{1,2\}) = 1$, and $w(x_i) = (x_i)^2$ we obtain

$\psi_v^w(31,49) = \psi_v^w(33,47) = \psi_v^w(41,31) = \psi_v^w(43,17) = 1537$ and $\psi_v^w(37,36) = 1354.4 < 1537$.



Compromise search, fairness or uncertainty aversion

Preference for interior points

$\forall x^1, \dots, x^n \in \mathbb{N}^n, \;\; \forall \alpha_1, \dots, \alpha_n \geq 0$ s.t. $\sum_{i=1}^{n} \alpha_i = 1$,

$$[x^1 \sim x^2 \sim \dots \sim x^n] \Rightarrow \sum_{i=1}^{n} \alpha_i x^i \succsim x^k, \quad k = 1, \dots, n$$

Proposition [Chateauneuf et Tallon, 02]

Within CED Theory, preference for interior points is equivalent to choosing w convex and v concave (minimization), where v is said to be concave iff :

$$v(A \cup B) + v(A \cap B) \leq v(A) + v(B) \text{ for all } A, B \subseteq N$$

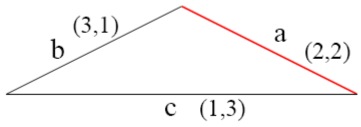
Complexity of Choquet optimization

If $v(A) = 1$ for all non-empty $A \subseteq Q$, then
$\psi_V^w(x) = \sum_{i=1}^{q} [w(x_{(i)}) - w(x_{(i-1)})]\, v(X_{(i)}) = w(\max_{i \in Q} x_i)$.
Hence the determination of a Choquet-optimal solution reduces to a min-max optimization problem.

- the min-max shortest path problem has been proved NP-hard by (Yu and Yang 98)
- the min-max spanning tree problem has been proved NP-hard by (Hamacher and Ruhe 94)

Consequently, the determination of a Choquet-optimal solution is NP-hard for both problems.

Failure of the greedy approach with Choquet



Choquet optimal edge: a (2, 2)

Completion: a ∪ b (5, 3) a ∪ c (3, 5) sub-optimal

b ∪ c is clearly better with (4, 4)

Idem for OWA, WOWA, Yaari's model, RDU, Lorenz, SSD…

An important notion: the core of a capacity

Dual capacity

$\bar{v}$ *dual capacity* of v if : $\forall A \subseteq S$, $\bar{v}(A) = 1 - v(S \setminus A)$

Core

- *core of a capacity* : with $\mathcal{L}$ the set of proba. on Q, $core(\bar{v}) = \{P \in \mathcal{L}, \bar{v}(A) \leq P(A) \leq v(A)\}$
- v concave $\Rightarrow$ $\bar{v}$ convex $\Rightarrow$ core($\bar{v}$)$\neq \emptyset$

Capacity in the core provide default approximations

Core of a capacity and associated weights

		$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{2,3\}$	$\{1,3\}$	N
	v	0	0.6	0.3	0.4	0.8	0.6	0.9	1
Shapley	ϕ	0	0.5	0.2	0.3	0.7	0.5	0.8	1
Max entropy	λ^*	0	0.4	0.3	0.3	0.7	0.6	0.7	1
	$\bar{v}$	0	0.4	0.1	0.2	0.6	0.4	0.7	1

p_1, p_2, p_3

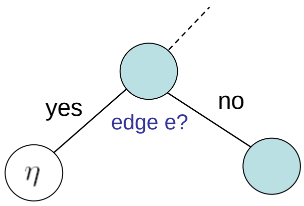
Proposition (lower bound)

$$\left.\begin{array}{l} v \text{ concave} \\ w \text{ convex} \end{array}\right\} \Rightarrow \forall P \in core(\bar{v}),\ \psi_v^w(x) \geq w(\sum_{i=1} p_i x_i)$$

(with $p_i = P(\{i\})$)

A1: Branch and Bound (spanning trees)

Requires a lower bound (must be easily computable)



1) Y set of admissible cost-vectors at node η
2) Find y^* in $\text{argmin}_{y \in Y} \sum_i p_i y_i$
3) Set $\mathbf{LB}(\eta) = w(\sum_i p_i y_i^*)$

2) Solved in polytime
2), 3) p chosen in the core

Improving bounds

1)
$$\max_{\lambda \in \mathbb{R}^n} f(\lambda) = \min_{y \in Y} \sum_{i=1}^{n} \lambda_i y_i,$$
$$\text{s.t. } \sum_{i \in A} \lambda_i \leq v(A) \quad \forall A \subseteq N,$$
$$\lambda_i \geq 0 \quad \forall i = 1, \ldots, n.$$

2) Find λ^* the optimal solution
3) Set $\mathbf{LB}(\eta) = w(f(\lambda^*))$

Numerical tests

TAB.: Branch&Bound approach : execution times (s)

		2 dim.		3 dim.		5 dim.		10 dim.	
		λ_i^*	ϕ_i	λ_i^*	ϕ_i	λ_i^*	ϕ_i	λ_i^*	ϕ_i
v_1	10	0	0	0.01	0.03	0.06	0.29	2.21	6.2
	15	0.01	0.11	0.23	9.45	2.41	804	36.8	>1h
	20	1.03	>1h	8.68	2726	31.4	>1h	>1h	>1h
	25	4.02	>1h	14.9	>1h	137.3	>1h	>1h	>1h
	30	13.4	>1h	60.7	>1h	>1h	>1h	>1h	>1h
v_2	10	0	0	0.01	0.03	0.1	0.11	4.23	12
	15	0.01	0.16	0.1	9.63	2.36	3.04	1950	1987
	20	0.48	40.13	0.86	63	72.1	>1h	>1h	>1h
	25	2.04	>1h	5.57	>1h	985.7	>1h	>1h	>1h
	30	5.11	>1h	48.6	>1h	3035	>1h	>1h	>1h

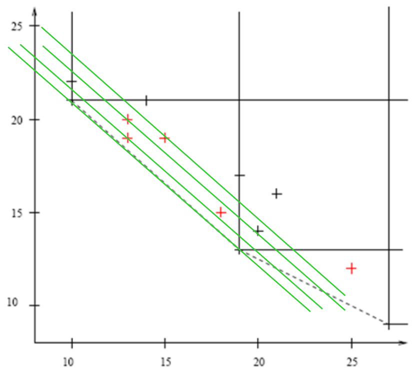
A2 :The ranking approach for ST

Linear scalarizing function :

$$f_\omega(x) = \sum_{i=1}^{q} \omega_i x_i$$

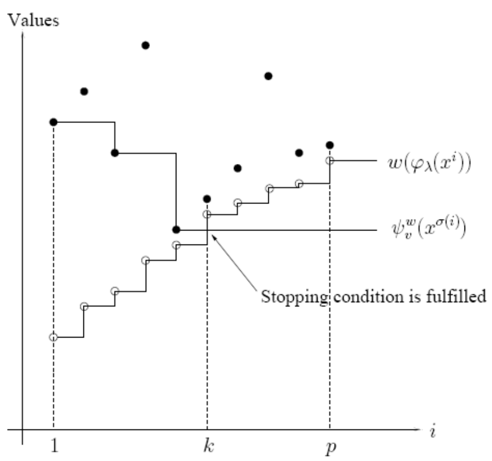
- optimization
+ ranking

- k-minimum spanning trees (Gabow, 77) or Ibaraki, Katoh, Mine (1981) : $O(km + min(n^2, m \log\log n))$

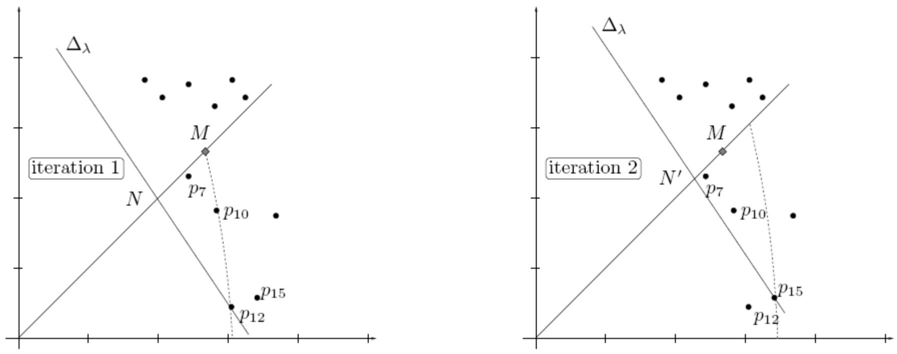


Requires a stopping conditions

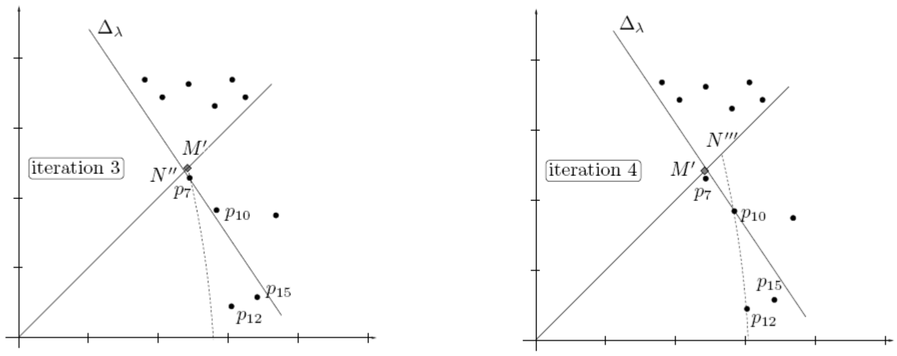
Stopping condition of the ranking approach



Example 1/2



Example 2/2



3. Approximation of Pareto-optimal Knapsacks

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths			1							
Trees										3
Flows							2			
Knapsack		4								
....										

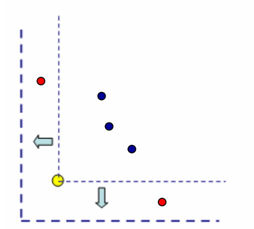
3. Approximation of preferred solutions

The case of Pareto dominance

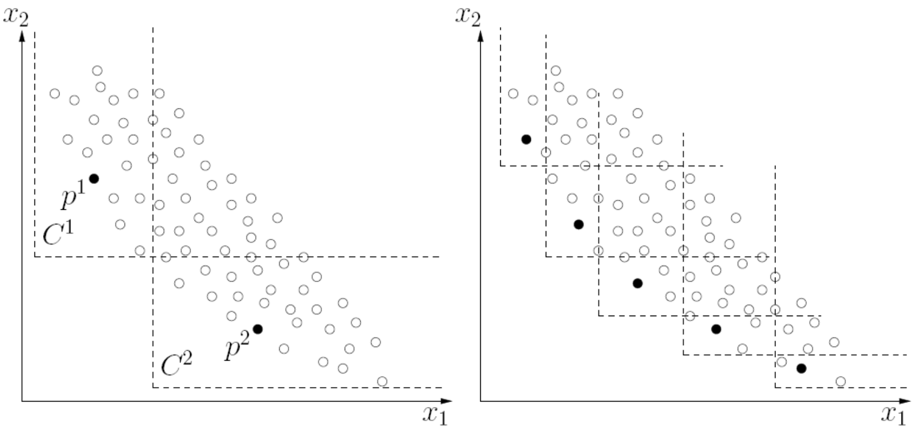
DEFINITION (ε-PARETO DOMINANCE)

Pour tout $\varepsilon > 0$ *et* $x, y \in X$,

$$x \succsim_\varepsilon y \Leftrightarrow [\forall i \in N, x_i \leq (1+\varepsilon)y_i]$$

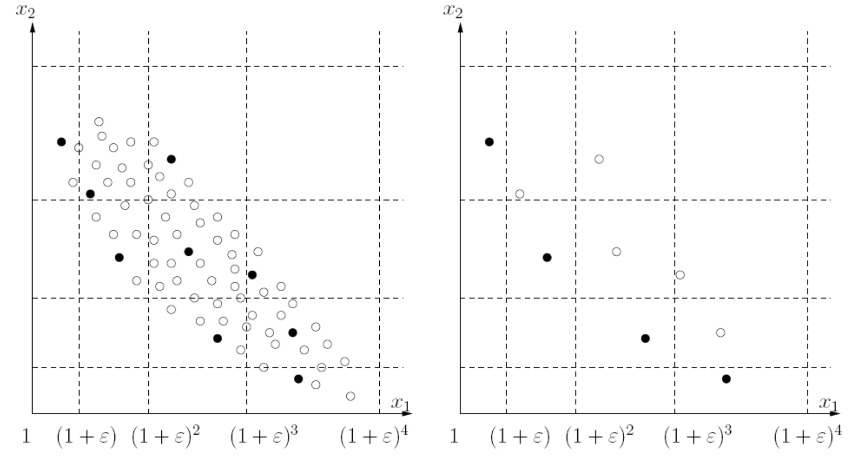


Approximation = covering of the Pareto set

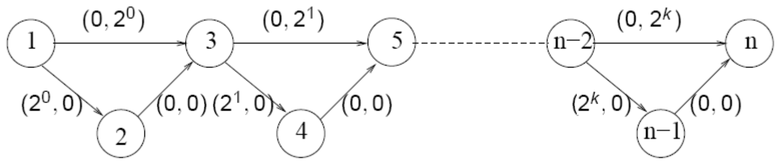


Existence of covering with bounded size (PY00)

$$(1 + \lfloor \log M / \log(1+\varepsilon) \rfloor)^m \qquad (1 + \lfloor \log M / \log(1+\varepsilon) \rfloor)^{m-1}$$



An example using Hansen's graphs



$$|P_\varepsilon| \leq \left\lfloor \frac{\log 2^{k+1}}{\log(1+\varepsilon)} \right\rfloor + 1$$

Example : $k = 15$ et $\varepsilon = 0.1$

- $2^{16} = 65536$ Pareto optimal elements
- $\left\lfloor \frac{\log 65536}{\log(1.1)} \right\rfloor + 1 = 117$ (upper bound)

Application to biobjective knapsack problems

$$\max \sum_{j=1}^{n} p_{1j} x_j, \quad \max \sum_{j=1}^{n} p_{2j} x_j$$
$$\text{subject to } \sum_{j=1}^{n} w_j x_j \leq b$$
$$x_j \in \{0,1\}\ \forall j \in \{1,\ldots,n\}$$

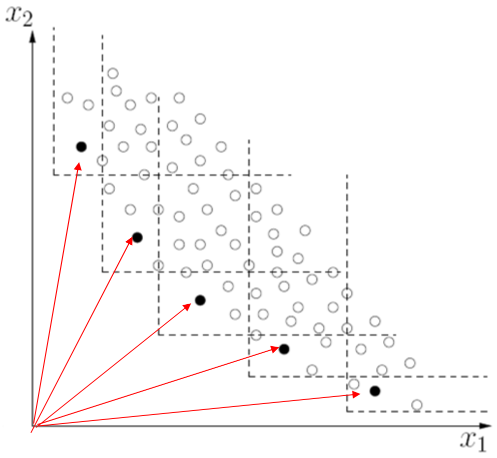
Project selection, product design, team configuration, resource allocation…

n	30	40	50	60	70	80
			MOA*			
time	0.397	1.879	11.31	43.66	215.2	457.7
ε			FPTAS			
0.005	0.353	1.514	7.922	29.90	127.8	226.5
0.01	0.297	1.077	4.842	18.29	65.91	97.26
0.05	0.046	0.036	0.065	0.331	0.555	0.393
0.1	0.003	0.001	0.001	0.002	0.001	0.001
ε			MOA^*_ε			
0.005	0.315	0.940	4.225	18.58	62.37	110.4
0.01	0.179	0.364	1.389	9.294	19.75	35.11
0.05	0.008	0.007	0.013	0.064	0.065	0.075
0.1	0.001	0.001	0.001	0.001	0.001	0.001

Table 1. Numerical results on the biobjective knapsack.

[Perny et Spanjaard, ECAI'08]

Approximation of preferred solutions for decision models refining Pareto dominance



Conclusion (main messages)

- Decision theory has provided various sophisticated models to go beyond Pareto dominance in different contexts
- Preference-based optimization with such models is significantly harder than in the classical case
- Classical approaches used for simple linear criteria and for Pareto Optimization do not extend easily (e.g. dynamic programming, greedy approaches)
- New problems requiring new algorithmic solutions
- Approximation of Pareto solutions with performance guarantees seems a good alternative
- Cross-fertilization of decision theory and (combinatorial) optimization, a challenging program for the future!

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Still some work to do…

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Recent publications of our team on this topic

Near Admissible Algorithms for Multiobjective Search
Perny, Patrice; Spanjaard, Olivier; ECAI-08 (2008) pp. 490-494

Search for Choquet-optimal paths under uncertainty
Galand, Lucie; Perny, Patrice, UAI'07, pp. 125-132,

State Space Search for Risk-averse Agents
Perny, Patrice; Spanjaard, Olivier; Storme, Louis-Xavier; IJCAI'07, pp. 2353-2358

A decision-theoretic approach to robust optimization in multivalued graphs
Perny, Patrice; Spanjaard, Olivier; Storme, Louis-Xavier;
Annals of Operations Research (2006) Vol. 147, 1, pp. 317-341

Search for Compromise Solutions in Multiobjective State Space Graphs
Galand, Lucie; Perny, Patrice; ECAI'06, pp. 93-97.

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