Collective Decision Making with Individual Confidence Scores in the Decision Rule

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Dimacs-Lamsade Workshop
Paris, 30-31 October 2008

Introduction and motivation

- Judgment aggregation: How can individual judgments (yes/no) on logically interconnected propositions be aggregated into corresponding collective judgments on the same propositions?
- Judgment aggregation focuses on a paradox of aggregation, the doctrinal paradox.
- The field is plagued by impossibility theorems.
- Motivation: A more realistic framework is needed.

Judgment Aggregation

- The problems investigated in this field are relevant and common to many situations: expert panels, legal courts, boards, councils, etc.
- *Example*: A candidate is offered a professor position in a university (conclusion R) only if he is good at teaching (premise P) and good at research (premise Q), that is the decision rule can be expressed as $(P \wedge Q) \leftrightarrow R$.
- In these situations, paradoxes can arise if one uses *propositionwise majority voting* as the aggregation procedure.
- To explore the space of possible aggregation procedures, various *impossibility theorems* have been proven.

The doctrinal paradox

- Back to our example: A committee has to decide whether a candidate should be offered a position (R). This is the case iff he is good at teaching (P) and at research (Q): $(P \wedge Q) \leftrightarrow R$. They vote as follows:

	P	Q	R
Member 1	Yes	Yes	Yes
Member 2	Yes	No	No
Member 3	No	Yes	No
Majority	Yes	Yes	No

- We learn: Propositionwise majority voting may lead to an inconsistency on the collective level.

Way out 1: The Premise-Based Procedure (PBP)

- Propositionswise majority voting on the premises, the infer the conclusion.
- *Intuition*: Reasons are important.

	P	Q	R
Member 1	Yes	Yes	–
Member 2	Yes	No	–
Member 3	No	Yes	–
Majority	Yes	Yes	Yes

- *Decision*: The candidate is offered the position.
- *Note*: This procedure is manipulable.

Way out 2: The Conclusion-Based Procedure (CBP)

- Propositionwise majority voting on the conclusion.
- *Intuition*: Committee members make up their minds on the premises privately and then submit their judgment on the conclusion.

	P	Q	R
Member 1	–	–	Yes
Member 2	–	–	No
Member 3	–	–	No
Majority	–	–	No

- *Decision*: The candidate is not offered the job.
- *Note 1*: This procedure is not manipulable.
- *Note 2*: The reasons for the decision cannot be given.

Way out 3: Distance-Based Procedures

- *Intuition*: Identify the situation that has minimal distance to the judgments expressed by the voters.
- *Advantage*: Find an aggregation method that ensures a consistent outcome and avoid dilemmas.
- How can this idea be made more precise?
- In our example $(P \wedge Q) \leftrightarrow R$, there are four admissible "judgment sets":

$$S_1 := (1,1,1) \quad , \quad S_2 := (1,0,0)$$
$$S_3 := (0,1,0) \quad , \quad S_4 := (0,0,0)$$

- One option is Hamming distance": distance between S_1 and S_4 is 3, between S_2 and S_3 is 2.
- *Limit*: This approach can avoids paradoxical outcomes at the price of indecision.

The classical paradox of judgment aggregation

A jury has to decide whether a defendant is liable for a breach of contract (R). According to legal doctrine, the defendant is liable if and only if the defendant did some action X (proposition P) and the defendant had a contractual obligation not to do action X (proposition Q).

	P	Q	R
Judge 1	1	1	1
Judge 2	1	0	0
Judge 3	0	1	0
Majority	1	1	0

PBP *vs.* CBP

- On the one hand, in a legal domain example, a premise-based procedure seems more appropriate. Indeed, the legal code requires the judges to provide arguments for their decision.
- On the other hand, in an example like the academic job offer, a conclusion-based procedure seems defendable since the agents are more affected by the final decision than by the reasons that supported that decision.

Impossibility theorems

List & Pettit (2002): There exists no aggregation procedure satisfying the following conditions:
Universal Domain (UD): Admissible inputs are any logically possible profile of individual sets of judgments. [A *profile* is an n-tuple $(J_1, J_2, \ldots, J_n)$ of individual judgment sets.]
Anonymity: All individuals have equal weight.
Systematicity: The collective judgment on each proposition depends only on the agent judgments on that proposition, and the aggregation rule is the same across all propositions.

In subsequent impossibility results, systematicity has been weakened to an independence condition:
Independence of Irrelevant Alternatives (IIA): IIA is systematicity without the neutrality condition, requiring that all propositions are equally treated.

Our approach - Aims

In our work we want:

1. To relax some of the assumptions made in the classical judgment aggregation framework for a more realistic approach.

2. To define an aggregation procedure that can turn either to PBP or CBP.

3. To attempt escaping the impossibility results in judgment aggregation problems while, at the same time, resolve indecision.

Our approach - Features

1. Judgment status:

Agents are allowed to be neutral on some criteria (i.e. premises) and to refuse to participate in the decision process (abstention).

2. Weighted *criteria*:

Group members can assign weights to the criteria in the decision rule (rule confidence scores).

Judgment status

We distinguish three possible judgments:

1. Classical binary judgment: 1 (for) or 0 (against)
2. Neutral judgment: Represented by "?", it means that the agent is unsure whether that criteria applies (1) or not (0). A group member may express a neutral judgment w.r.t. some or all criteria (and - possibly - on the conclusion as well).
3. Abstention: An agent does not give any judgment on the propositions in the decision rule. Abstainers are not taken into account in the aggregation process but in the computation of the *legitimacy* of the decision.

Legitimacy

The legitimacy (lg) is equal to the total number of voters over the number of authorized people to vote (i.e $0 \leq lg \leq 1$). The closer lg is to 1, more support the process received from the individuals entitled to participate in it. The legitimacy does not play a role in the final outcome. However, the legitimacy level may declare the decision outcome invalid.

Confidence in the decision rule (1)

- A decision rule has the form $(P_1 \wedge \cdots \wedge P_n) \leftrightarrow R$, where P_i are criteria and R is the conclusion.
- A weight $\alpha_{ij} \in [0, 1]$ is associated to each criterion P_i: it expresses how much member j deems P_i to be relevant for the conclusion.
- $0 \leq \alpha_{1j} + \cdots + \alpha_{nj} \leq 1$
- When $\alpha_{ij} = 0$, the judgment corresponding to the associated criterion P_i is ignored and the value of R is decided only using the remaining criteria.

Confidence in the decision rule (2)

We distinguish the following cases:

- Full agreement ($\alpha_{1j} + \cdots + \alpha_{nj} = 1$): For member j, either the criteria $P_1, \cdots, P_n$ are the all and only relevant ones to make a judgment on R, or they *include* all the relevant criteria together with some completely irrelevant ones. Thus j completely agrees on the decision rule (e.g. jury).
- Partial agreement ($\alpha_{1j} + \cdots + \alpha_{nj} < 1$): Member j doesn't fully agree on the decision rule, i.e. he deems that (all or some of) the relevant criteria have been dismissed (and, possibly, that the rule includes some irrelevant criteria for the decision).
  - High partial agreement: $t \leq \alpha_{1j} + \cdots + \alpha_{nj} < 1$
  - Low partial agreement: $0 \leq \alpha_{1j} + \cdots + \alpha_{nj} < t$

where t is a prefixed threshold.

Confidence in the decision rule - Example (1)

The members of a board of a research funding agency have to decide which research project to support (R) on the basis of three criteria: quality (P), originality (Q), and applicability (S). Suppose that the applicability criterion has been introduced only recently following some new regulation that impose all research funding agency to be evaluated on the basis of likeness to attract the interest of private funding. If a good part of the board members dissent with the criterion S, they will cast their votes on the propositions, but assign a very low weight to S. This will be reflected at the end of the process, when a certain decision will be made, but also the information about how the group views the criteria selected for the rule will be publicly available.

Confidence in the decision rule - Example (2)

The five members of the board state their judgments as below:

	P	Q	S
M_1	(0, .33)	(0, .33)	(1, .34)
M_2	(1, .3)	(1, .3)	(1, .4)
M_3	(0, .5)	(0, .5)	(0, 0)
M_4	(1, .3)	(1, .3)	(0, 0)
M_5	(1, .2)	(1, .1)	(1, .1)

The criteria weights should play a role in the way group members express their judgments on the conclusion. Moreover, the information about how relevant the members deem the criteria to be for the decision has to be taken into account when the individual judgments are aggregated.

Confidence in the decision rule (3)

How do individuals assess R given the criteria weights?

- High partial agreement: $t \leq \alpha_{1j} + \cdots + \alpha_{nj} < 1$: Even if j does not fully agree on the decision rule, he believes that this includes enough relevant criteria ($\alpha_{1j} + \cdots + \alpha_{nj} \geq t$). The judgment on the conclusion is obtained following the decision rule.
- Low partial agreement: $0 \leq \alpha_{1j} + \cdots + \alpha_{nj} < t$: The confidence in the decision rule is very low, i.e. criteria P_i are not adequate or some very important criteria are missing. So j fixes the value of R according also to the missing criteria. The decision rule for j is $(P_1 \wedge \cdots \wedge P_m \wedge T_1 \wedge \cdots \wedge T_l) \leftrightarrow R$.

Confidence in the decision rule - Example (3)

The weights are used to compute the confidence score CS_j of the rule for each group member j; namely $CS_j = \alpha_{1j} + \cdots + \alpha_{nj}$.

Suppose that $t = .5$

	P	Q	S	CS	R
M_1	(0, .33)	(0, .33)	(1, .34)	1	0
M_2	(1, .3)	(1, .3)	(1, .4)	1	1
M_3	(0, .5)	(0, .5)	(0, 0)	1	0
M_4	(1, .3)	(1, .3)	(0, 0)	.6	1
M_5	(1, .2)	(1, .1)	(1, .1)	.4	0

Aggregation procedure (1)

How is the collective decision derived?

$D = ((P_{agg_1}, \alpha_{agg_1}), \cdots, (P_{agg_n}, \alpha_{agg_n}), R_{agg}, CS_{agg}, lg)$

- P_{agg_i} is the proposition-wise majority voting on $P_{i1}, \cdots, P_{ik}$ (with $\alpha_{ij} \neq 0$). Neutral judgments simply follow the majority. In case of a tie, compute the sum of α_{ij} associated to $P_{ij} = 1$ and the sum of α_{ij} associated to $P_{ij} = 0$ taken individually and follow the judgment corresponding to the greatest sum.
- α_{agg_i} (resp. CS_{agg}) is the average function of $\alpha_{i1}, \cdots, \alpha_{ik}$ (resp. $CS_1, \cdots, CS_k$). Note that $CS_{agg} = \alpha_{agg_1} + \cdots + \alpha_{agg_n}$.

Aggregation procedure (2)

$$D = ((P_{agg_1}, \alpha_{agg_1}), \cdots, (P_{agg_n}, \alpha_{agg_n}), R_{agg}, CS_{agg}, lg)$$

- R_{agg} is computed by PBP or CBP. The procedure is chosen w.r.t. CS_{agg} and t:
  - If $CS_{agg} < t$ then we use CBP and R is calculated by simple majority voting on $R_1, \cdots, R_k$. $CS_{agg} < t$ means that the members deem the decision rule not the right one, so the only reasonable thing they can say is the final conclusion, without giving reasons for that.
  - If $CS_{agg} \geq t$ then R_{agg} is computed by PBP.
- lg is the legitimacy. It is equal to the total number of voters over the number of authorized people to vote.

Aggregation procedure - Example (1)

Example of judgment aggregation with high confidence score ($t = .8$) $\Rightarrow$ PBP

	P	Q	S	CS	R
M_1	(1, .5)	(0, .5)	(?, 0)	1	0
M_2	(?, .4)	(1, .4)	(1, .1)	.9	?
M_3	X	X	X	X	X
M_4	(1, .3)	(1, .4)	(?, .1)	.8	?
M_5	(1, .4)	(1, .3)	(1, .05)	.75	1
collective decision	(1, .4)	(1, .4)	(1, .06)	.86	1

Aggregation procedure - Example (2)

Example of judgment aggregation with low confidence score $(t = .5) \Rightarrow$ *CBP*

	P	Q	CS	R
M_1	(1, .1)	(1, .2)	.3	0
M_2	(0, .1)	(1, .1)	.2	0
M_3	(1, .2)	(0, .2)	.4	0
M_4	(1, .2)	(1, .1)	.3	1
M_5	(0, .2)	(0, .2)	.4	0
collective decision	(1, .16)	(1, .16)	.32	0

Conclusion and future work

- Our approach aims at relaxing two assumptions made in the classical judgment aggregation framework:
  - Individuals are required to express 0/1 judgments on all propositions in the agenda.
  - They have to be fully committed to the decision rule.
- We define an aggregation procedure that can turn either to PBP or CBP.
- In future work our goal is to study the properties that our approach satisfies and
- to investigate to which extent it provides an escape from the impossibility results.