Collective decision-making with individual confidence scores in the decision rule

Farah Benamara*, Souhila Kaci†, Gabriella Pigozzi*

Abstract

Judgment aggregation (JA) studies how to aggregate individual judgments to form collective decisions. Examples are expert panels, legal courts, boards, and councils. The problems investigated in this new field are relevant and common to many situations. Nevertheless, the existing procedures are idealized and, likewise the related problems of preference aggregation in social choice theory, the field is plagued by impossibility theorems. In this paper, we extend standard JA in order to offer a more realistic framework and to escape the impossibility results. We propose to distinguish between abstainers and neutral judgment as well as to model the notion of confidence a group member may have in the decision rule by assigning to each criterion a normalized weight. We then show how this new framework may help us to avoid indecision in most cases.

Key words : Judgment aggregation, group decision-making

Abstract

Judgment aggregation (JA) studies how to aggregate individual judgments to form collective decisions. Nevertheless, the existing procedures are idealized and the field is plagued by impossibility theorems. In this paper, we extend standard JA in order to offer a more realistic framework and to escape the impossibility results.

*IRIT-CNRS, 118 rt de Narbonne, 31062, Toulouse, France. {benamara}@irit.fr

†CRIL-CNRS, Rue de l'Université SP 16, 62307, Lens, France {kaci}@cril.univ-artois.fr

‡University of Luxembourg, Individual and Collective Reasoning, Computer Science and Communication, Luxembourg. {gabriella.pigozzi}@uni.lu

1 Introduction

Judgment aggregation (JA) is a recent formal discipline that studies how to aggregate individual judgments to form collective decisions. Examples are expert panels, legal courts, boards, and councils This field has recently attracted attention in multi-agent systems and artificial intelligence, in particular due to the relations with belief merging [2], for example for the combination of opinions of equally reliable agents. JA problems consider a group of people stating their views (in the binary form of 1 or 0) on some logically interconnected propositions. An example is the problem of choosing a candidate for a professor position [3]. A candidate is offered the job (*conclusion* R) only if she is good at teaching (*premise* P) and good at research (*premise* Q), that is the decision rule can be expressed as $(P \wedge Q) \leftrightarrow R$. As we will see, problems arise because seemingly reasonable aggregation procedure leads to paradoxical outcomes.

Clearly, the problems investigated in this new field are relevant and common to many situations. Nevertheless, the existing procedures are idealized and, likewise the related problems of preference aggregation in social choice theory [4], the field is plagued by impossibility theorems. To provide a more realistic framework, and to escape the impossibility results are among the goals of the paper. More specifically, we introduce the following changes:

1. An agent may not vote and thus abstain. In the previous example, a committee member may abstain because she believes that the decision rule is inappropriate, or because there is no suitable candidate.

2. It is not realistic to impose that the agents always have a clear position on every proposition. Our model allows the individuals to express a neutral judgment. It is worth noticing that abstention and neutral judgments are distinct. The difference will be clarified later in the paper.

3. Borrowing the terminology from the field of multiple-criteria decision making [6], we call the propositions that support a certain conclusion *criteria* (instead of premises). This is justified by the fact that, in many decision problems, agents make their evaluations by taking into account different criteria and, even when the individuals agree on the criteria, they may assign them different weights. For example, some agents may deem research to be more important than teaching. Moreover, we allow the group members to state whether or not they agree on the rule governing the decision.

4. Some procedures can avoid paradoxical outcomes at the price of indecision [2]. However, indecision is a very tedious problem. Our more refined procedure is an attempt to escape the impossibility results in JA problems while, at the same time, to resolve indecision in most cases.

In this paper, we propose to extend standard JA to take into account the above considerations. More precisely, we will answer the following research questions:

- How to model *judgment status*? We will distinguish three cases: (a) classical binary evaluations, (b) neutral judgments and (c) abstentions.
- How to model *the confidence a member has in the decision rule*? We propose to model the notion of confidence by assigning to each criterion a normalized weight.
- How to adapt standard aggregation procedures to take into account the judgment status as well as the confidence? We then show how this additional information may help us to avoid indecision in most cases.
- How to model *degree of support*? We propose the notion of *legitimacy*.

The paper is organized as follows. After necessary background on the problem of JA, we first recall some related works. We then present our general framework extending classical JA procedure with confidence in the decision rule, judgment status and legitimacy of the result. After that, we introduce the formal representation, the aggregation procedure and some properties of our framework.

2 Judgment aggregation

In the original problem of JA [8], a court has to make a decision on whether a person is liable of breaching a contract (proposition R). The judges have to reach a verdict following the legal doctrine. This states that a person is liable if and only if there was a contract (P) and there was a conduct constituting breach of such a contract (Q). The legal doctrine can be formally expressed by the rule $(P \wedge Q) \leftrightarrow R$. Each member of the court expresses her judgment on the propositions P, Q and R such that the rule $(P \wedge Q) \leftrightarrow R$ is satisfied.

	P	Q	$R = (P \wedge Q)$
Judge A	1	0	0
Judge B	0	1	0
Judge C	1	1	1
Majority	1	1	0

Table 1: Doctrinal paradox. Premises: P = There was a contract, Q = There was conduct constituting breach of such a contract. Conclusion: R = There was a breach of contract.

Suppose now that the three members of the court make their judgments according to Table 1. Each judge expresses a consistent opinion, i.e. she says yes to R if and only if she says yes to both P and Q. However, *proposition-wise* majority voting (consisting in the separate aggregation of the votes for each proposition P, Q and R via majority rule) results in a majority for P and Q and yet a majority for $\neg R$. This is an inconsistent collective result, in the sense that $\{P, Q, \neg R, (P \wedge Q) \leftrightarrow R\}$ is inconsistent in propositional logic. The paradox lies in the fact that majority voting can lead a group of rational agents to endorse an irrational collective judgment, i.e. to have a majority believing that the defendant should be left free while *another* majority deems there are reasons to sentence her. The literature on JA refers to such problem as the *doctrinal paradox*. Clearly, the relevance of such aggregation problems goes beyond the specific court example and affects all collective decisions on logically interconnected propositions.

The first two ways to avoid the inconsistency that have been suggested are the *premise-based procedure* (henceforth, PBP) and the *conclusion-based procedure* (henceforth, CBP) According to the PBP, each member casts her vote on each premise. The conclusion is then inferred from the judgment of the majority of the group on the premises using the rule $(P \wedge Q) \leftrightarrow R$. In the example above, the PBP would declare the defendant liable.

According to the CBP, the members decide privately on P and Q and only express their opinions on R publicly. The judgment of the group is then inferred from applying the majority rule to the agents' judgments on the conclusion. No reasons for the court decision can be supplied. Contrary to the PBP, the application of the CBP would free the defendant. In order to investigate how strong the paradoxical outcomes are, some seemingly reasonable conditions were assumed on the aggregation function. Unfortunately, most of the results obtained in the field are negative [12, 13, 14, 15].

To give a flavor of a typical impossibility result in the JA field, and to introduce some terminology, we state the first impossibility theorem [12]. A set of agents $N = \{1, 2, \ldots, n\}$, with $n \geq 3$, has to make judgments on logically interconnected propositions of a language $\mathcal{L}$. The set of propositions on which the judgments have to be made is called *agenda*. A (individual or collective) *judgment set* is the set of propositions believed by the agents or the group. An n-tuple $(J_1, J_2, \ldots, J_n)$ of agent judgment sets is called *profile*. A *judgment aggregation rule* F assigns a collective judgment set J to each profile $(J_1, J_2, \ldots, J_n)$ of agent judgment sets. A judgment set is *consistent* if it is a consistent set in $\mathcal{L}$, and is *complete* if, for any $P \in \mathcal{L}$, $P \in J$ or $\neg P \in J$.

A set of seemingly rational and desirable conditions are imposed on the aggregation rules and then, typically, an impossibility result is derived. The first impossibility theorem of JA states that there exists no aggregation rule F satisfying the following conditions:

Universal Domain: The domain of F is the set of all profiles of consistent and complete judgment sets.

Anonymity: Intuitively, this means that all agents have equal weight.

Systematicity: This condition ensures that the collective judgment on each proposition depends only on the agent judgments on that proposition, and that the aggregation rule is the same across all propositions. Systematicity is clearly a very strong condition. In subsequent impossibility results, systematicity has been weakened to the independence of irrelevant alternatives (IIA):

IIA: It is systematicity without the neutrality condition, requiring that all propositions are equally treated.

3 Related works

In this section we refer to works that proposed to relax some of the assumptions made in the classical JA framework. However, our model is the first that combines all these different aspects and introduces new ones.

3.1 Abstention and neutral judgments

Results in JA usually assume complete judgment sets both at the individual and collective level. Gärdenfors [16] was the first to criticize such assumption as being too strong and unrealistic. He allows voters to abstain from expressing judgments on some propositions in the agenda. He proves that, if the judgment sets may not be complete (but logically closed and consistent), then every aggregation function that is IIA and Paretian[3] must be oligarchic.[4] Gärdenfors' framework requires the agenda to have a very rich logical structure (with an infinite number of issues). More recently, Dokow and Holzman [17] extended Gärdenfors' result and consider finite agendas. Again, impossibility results are obtained. Hence, relaxing the completeness assumption does not avoid the impossibility results.

Nevertheless, allowing the voters to not express their judgments on some of the issues in the agenda provides a more realistic model of JA, which is the aim of our paper. In order to avoid confusion, we must observe that we distinguish abstaining from being neutral with respect to an issue in the agenda. Abstentions in Gärdenfors and Dokow and Holzman' works correspond to what we call "neutral judgments". In our model, a voter abstains when she does not state her judgments on *any* issue in the agenda. Abstaining is a meaningful position, i.e. the refuse to participate to the decision process. This will

[3] A *Paretian* aggregation function is such that, if all the individuals in the group adopt the same position on a certain issue, this position will be adopted at the collective level as well.

[4] An aggregation function is *oligarchic* if, for every issue in the agenda, the group adopts a position 0 (resp. 1) if and only if all the members of a subset of the group (the oligarchy) adopt position 0 (resp. 1) on that issue. Clearly, when there is only one individual in the oligarchy, it corresponds to dictatorship.

affect the *legitimacy* of the decision outcome as an election with a high abstention rate is invalid.

On the other hand, being neutral on a certain issue captures those situations in which voters do not have a clear position on that issue, do not feel competent, or simply prefer not to take position on that matter. Unlike the abstainers, these voters wish to actively participate to the decision process. For example, given $(P \wedge Q) \leftrightarrow R$, if an individual believes P to be true but does not know about Q, then her judgment set will be $\{(1,0,0),(1,1,1)\}$.

3.2 Weighted criteria

Borrowing the terminology from multi-criteria decision making, we will refer to the premises as *criteria* for the decision. Likewise multiple criteria decision methods, group members in a JA setting evaluate a candidate on the basis of a finite number of premises. However, the two fields have some differences. The first is that multi-criteria decision methods study how a group can select (possibly) one candidate from a set of alternatives by attributing weights to each criterion. Instead, in JA, group members are asked to express their judgments on propositions that refer to a single candidate per time. The second noticeable difference is in the distinction between criteria/premises and conclusion in JA, which is missing in multi-criteria literature, as well as the logical interrelations among those propositions.

The main novelty we introduce in the JA framework is that in our model group members assign *weights* to the criteria in the decision rule. As we will see, this amounts to allocate a *confidence score* to the rule governing the decision. Group members not only participate to the decision process but they also express how well-suited the adopted rule is for the decision at hand.

So far the only approach that resembles ours is that of Dietrich and List [18], where they investigate JA problems using quota rules. A threshold is fixed for each proposition in the agenda and a proposition is collectively accepted only if the number of group members accepting it is at least equal to the threshold for that proposition. In many real-world decision-making situations, the propositions may differ in status and importance, which is taken into consideration by fixing different threshold values. We sympathize with that approach but we want to go further. Instead of having an a priori fixed threshold for accepting each proposition, we want to allow the group members to state their individual views on the relevance of each criterion to the decision. In this way, people can also express how much they agree with (resp. dissent from) the rule, by assigning high (resp. low) weights to the criteria. The closer the sum of the weights is to 1, the more an individual believes that the rule is the correct one to assess that decision problem. If, on the other hand, she assigns low weights, it means that she deems that the more important criteria have been overlooked or dismissed.

As an example, consider the board of a research funding agency whose members have to decide which research project to support on the basis of three criteria: quality (P), originality (Q), and applicability (S). Assume as well that the applicability criterion has been introduced only recently following some new regulation that impose all research funding agency to be evaluated on the basis of likeness to attract the interest of private funding. If a good part of the board members dissent with the criterion S because they believe that this will damage pure-theoretical projects to the benefit of pure applied ones, they will cast their votes on the propositions, but assign a very low weight to S. This will be reflected at the end of the process, when a certain decision will be made, but also the information about how the group views the criteria selected for the rule will be publicly available.

4 General Framework

Without loss of generality[5], any formula modeling JA can be written under the following form

$$(P_1 \wedge \cdots \wedge P_n) \leftrightarrow R \tag{1}$$

where P_i are criteria and R is the conclusion. In the following (1) is referred to as the *decision rule*. Let us now formalize the extensions we intend to give to classical judgments aggregation, namely: *confidence in the decision rule*, *judgment status* and *legitimacy of the result*.

4.1 Confidence in the decision rule

The confidence in the decision rule is represented locally at the level of each criterion. A weight $\alpha_{ij} \in [0, 1]$ associated to a criterion P_i expresses how much P_i is relevant for the conclusion in the decision rule for the member j[6]. Thus (1) is generalized as follows:

$$(P_1, \alpha_{1j})\Lambda \cdots \Lambda(P_n, \alpha_{nj}) \text{ iff } R, \tag{2}$$

with $0 \leq \alpha_{1j} + \cdots + \alpha_{nj} \leq 1$, where Λ stands for the conjunction between weighted criteria. Note that, when $\alpha_{ij} = 0$ the judgment corresponding to the associated criterion P_i is simply ignored and the value of R is decided only using the remaining criteria. This

[5] Since any propositional formula P can be written in a disjunctive form, i.e. $P = (P_1 \wedge \cdots \wedge P_n) \vee (P'_1 \wedge \cdots \wedge P'_m) \vee \cdots$ then handling $P \leftrightarrow R$ turns to handling $P_1 \wedge \cdots \wedge P_n \leftrightarrow R$ or $P'_1 \wedge \cdots \wedge P'_m \leftrightarrow R$ or .... Indeed we describe the JA procedure corresponding to $P_1 \wedge \cdots \wedge P_n \leftrightarrow R$. The other decision rules $P'_1 \wedge \cdots \wedge P'_m \leftrightarrow R$, ... are treated in a similar way.

[6] It is important to notice that α_{ij} does not express how much a member is confident when expressing her judgment w.r.t. a criterion P_i, but how much the member judges P_i relevant in the decision rule.

is intuitively meaningful since $\alpha_{ij} = 0$ means that the member j judges that the criterion P_i should not be considered. Depending on the values of the weights $\alpha_{1j}, \cdots, \alpha_{nj}$, we distinguish the following cases:

a) Full agreement "$\alpha_{1j} + \cdots + \alpha_{nj} = 1$". This means that, for member j, either the criteria $P_1, \cdots, P_n$ are the all and only relevant ones to make a judgment on R, or they *include* all the relevant criteria together with some completely irrelevant ones. Thus j completely agrees on (1). Indeed (2) reduces to (1) such that P_i is ignored if $\alpha_{ij} = 0$, i.e. the decision rule is $(P_1 \wedge \cdots \wedge P_{i-1} \wedge P_{i+1} \wedge \cdots \wedge P_n) \leftrightarrow R$.
It is worth observing that the original legal paradox of judgment aggregation is an instance of the full agreement case, where all group members (the judges) have to fully endorse the legal code, or behave as if this is the case.

b) Partial agreement "$\alpha_{1j} + \cdots + \alpha_{nj} < 1$". This case means that the member j doesn't fully agree on the decision rule, i.e. she deems that (all or some of) the relevant criteria have been dismissed (and, eventually, that the rule includes some irrelevant criteria for the decision).

Example 1 *Let us consider the example of the board of a research funding agency described above. We recall that the decision on whether to support a research project is taken by looking at three criteria: quality (P), originality (Q), and applicability (S). The five members state their judgments on P, Q and S as in Table 2.*

	P	Q	S
M_1	(0, .33)	(0, .33)	(1, .34)
M_2	(1, .3)	(1, .3)	(1, .4)
M_3	(0, .5)	(0, .5)	(0, 0)
M_4	(1, .3)	(1, .3)	(0, 0)
M_5	(1, .2)	(1, .1)	(1, .1)

Table 2: Individual judgments and weights assignments on the criteria.

The first two members deem the criteria P, Q, and S to be the all and only relevant attributes for funding a project. Since for them $\alpha_{1j} + \alpha_{2j} + \alpha_{3j} = 1$ ($j = 1, 2$), they fully agree with the decision rule. The third member also fully agrees with the rule but, unlike the first two, she believes that P and Q are the only relevant criteria and S is completely irrelevant for the decision. Like M_3, M_4 thinks that the applicability criterion should not play a role in the decision of which project to fund. However, she believes that one or more relevant criteria have not been taken into consideration ($\alpha_{14} + \alpha_{24} + \alpha_{34} < 1$). Finally, M_5 agrees with M_4 that other important criteria for the decision have been ignored, but she thinks that the applicability of a project should be taken into account, though it is not a very important aspect for the final decision.

The criteria weights should play a role in the way group members express their judgments on the conclusion. Moreover, the information about how relevant the members deem the criteria to be for the decision has to be taken into account when the individual judgments are aggregated to derive a collective decision. We distinguish the following sub-cases depending on the level of global non-agreement on the decision rule. Let t $(1 > t > 0)$ be a threshold.

b.1) High partial agreement "$t \leq \alpha_{1j} + \cdots + \alpha_{nj} < 1$". Even if member j does not fully agree on the decision rule, she believes that this includes enough relevant criteria $(\alpha_{1j} + \cdots + \alpha_{nj} \geq t)$. This means that j deems the rule sufficiently appropriate to decide R on the basis of the given criteria. Hence, the judgment on the conclusion is obtained following the given rule: (2) reduces to (1) such that P_i is ignored if $\alpha_{ij} = 0$.
Note that **a** and **b.1** can be both represented by $t \leq \alpha_{1j} + \cdots + \alpha_{nj} \leq 1$.

b.2) Low partial agreement "$0 \leq \alpha_{1j} + \cdots + \alpha_{nj} < t$". In this case, the confidence in the decision rule is very low, i.e. criteria P_i are not adequate or some very important criteria are missing. In this case, the member fixes the value of R according also to the missing criteria. The intuition is that, if an individual wants to have her saying in a decision process, but considers the adopted rule unable to capture the relevant criteria for the decision, she must be able to express her judgment on the conclusion while making explicit that she deems the rule to be not completely appropriate. Note, however that if a group member assigns a judgment 0 to a criterion P_i and $\alpha_{ij} \neq 0$ then R should be 0. This is to ensure coherence. Indeed the decision rule for that member is $(P_1 \wedge \cdots \wedge P_m \wedge T_1 \wedge \cdots \wedge T_l) \leftrightarrow R$, where $P_1, \cdots, P_m$ are criteria whose associated α_{ij} is different from 0 and $T_1, \cdots, T_l$ are missing criteria. Since only criteria P_i are present in the decision process, we may for example have $R = 0$ while $P_1 = \cdots = P_m = 1$[7] because the judgment of the group member j is 0 w.r.t. at least one missing criterion. Now if at least one P_i has a judgment 0 then R is necessarily equal to 0. Indeed a group member is free to fix the value of R only in case $0 \leq \alpha_{1j} + \cdots + \alpha_{nj} < t$ and all criteria whose associated weight is different from 0 are assigned a judgment 1.

The weights $\alpha_{1j}, \cdots, \alpha_{nj}$ are then used to compute the confidence score CS_j of the rule for each group member j; namely $CS_j = \alpha_{1j} + \cdots + \alpha_{nj}$.

Example 2 *Following* **b.1** *and* **b.2***, the table below summarizes the judgments of the members of our funding board example:*

The first three members fully agree with the decision rule $(P \wedge Q \wedge S) \leftrightarrow R$. Hence, the judgment on the conclusion R is logically derived from the values assigned to P, Q, S and the rule. Things are different for the last two group members. Suppose that $t = .5$. M_4 assigned zero weight to S but her CS is above the threshold. Hence, M_4 assigns R a value according to the values of only P and Q and the decision rule. Instead, M_5 has

[7] $P_i = 1$ is a short hand notation to express that the judgment w.r.t. criterion P_i is 1.

	P	Q	S	CS	R
M_1	(0, .33)	(0, .33)	(1, .34)	1	0
M_2	(1, .3)	(1, .3)	(1, .4)	1	1
M_3	(0, .5)	(0, .5)	(0, 0)	1	0
M_4	(1, .3)	(1, .3)	(0, 0)	.6	1
M_5	(1, .2)	(1, .1)	(1, .1)	.4	0

Table 3: Individual judgments on all propositions in the agenda.

a low CS in the rule. In order to capture the intuition that she should cast her vote on R sincerely (according to what she considers the missing criteria), M_5 can decide whether the research project without following the rule. For instance, she can refuse the funding ($R = 0$).

When $CS_j = \alpha_{1j} + \cdots + \alpha_{nj} = 0$ the judgments of the group member j are ignored but this is not considered as an abstention since the given α_{ij} are considered in the aggregation process, as explained in the next section.

4.2 Judgment status

We distinguish three possible judgments: classical binary judgment 1 (for) or 0 (against), neutral judgment and abstention. As classical binary judgment is already used, we only detail abstention and neutral judgments.

- We represent a **neutral judgment** by a question mark "?" i.e the judgment may be 1 or 0. A group member may express a neutral judgment w.r.t. some or all criteria (and - eventually - on the conclusion as well).
- In case of **abstention**, a group member does not give any judgment on $P_1, \cdots, P_n$ (and by consequence no values on $\alpha_1, \cdots, \alpha_n$) and R. Abstainers are not taken into account in the aggregation process

4.3 Legitimacy

The legitimacy, denoted lg, expresses to what extent the decision process is reliable. It is equal to the total number of voters over the number of authorized people to vote (i.e $0 \leq lg \leq 1$). The closer lg is to 1, more reliable the process is. The legitimacy does not play a role in the final outcome. So the legitimacy level may declare the decision outcome invalid. But this comes in a second step. First we aggregate using our aggregation procedure that we will present in the next section and then legitimacy considerations can play a role.

5 Representation and aggregation procedure

We represent a judgment expressed by a member j by the following tuple

$$J_j = ((P_{1j}, \alpha_{1j}), \cdots, (P_{nj}, \alpha_{nj}), R_j, CS_j),$$

$P_{ij}, R_j \in \{0, 1, ?\}$ and $\alpha_{ij}, CS_j \in [0, 1]$. Note that CS_j can be computed from $\alpha_{1j}, \cdots \alpha_{nj}$ (we have $CS_j = \alpha_{1j} + \cdots + \alpha_{nj}$). However we include CS_j in J_j for simplicity reading since it allows to see whether we apply the decision rule or not for member j. This remark also holds for CS_{agg} in D given below. R_j is either derived following the decision rule or fixed by the group member depending on whether the confidence rule CS_j is above the threshold or not. In case of abstention we write $J_j = (X, \cdots, X, X, X)$. Given a set of judgments $\{J_1, \cdots, J_k\}$, the collective decision is represented as follows:

$$D = ((P_{agg_1}, \alpha_{agg_1}), \cdots, (P_{agg_n}, \alpha_{agg_n}), R_{agg}, CS_{agg}, lg),$$

- P_{agg_i} is the majority of $P_{i1}, \cdots, P_{ik}$ (with $\alpha_{ij} \neq 0$) following proposition-wise majority voting. So neutral judgments simply follow the majority. In case of a tie between the number of $P = 1$ and $P = 0$, compute the sum of α_{ij} associated to $P_{ij} = 1$ and the sum of α_{ij} associated to $P_{ij} = 0$ taken individually and follow the judgment corresponding to the greatest sum. In this case, neutral judgments follow the will of judgments having a greater weight. In case of a tie, we put $P_{agg_i} =?$. Note that this is the only extreme case where our approach does not solve the indecision.

- α_{agg_i} (resp. CS_{agg}) is a numerical aggregation of $\alpha_{i1}, \cdots, \alpha_{ik}$ (resp. $CS_1, \cdots, CS_k$). In this paper, we use the average function but any other numerical aggregation function may be used as well. Note that $CS_{agg} = \alpha_{agg_1} + \cdots + \alpha_{agg_n}$. This is important since it means that expressing the confidence in the decision rule or relevance of each criterion leads to the same result.

- R_{agg} is computed by PBP or CBP. The procedure is chosen w.r.t. CS_{agg}:

  - if $CS_{agg} < t$ then we use CBP and R is computed on the basis of $R_1, \cdots, R_k$. This is intuitively meaningful since $CS_{agg} < t$ means that the group members thought that the decision rule was not the right one for that decision, so the only reasonable thing they can say is the final conclusion, without giving reasons for that. R_{agg} is calculated by simple majority voting. In case of indecision we compute the sum of CS_j for which $R_j = 1$ and the sum of CS_j for which $R_j = 0$. Then we follow the judgment associated to the greatest sum. In case of a tie, we put $R_{agg} =?$. Again this is the only extreme case where indecision is not solved.

  - if $CS_{agg} \geq t$ then R_{agg} is computed following PBP. In fact $CS_{agg} \geq t$ means that we agree on the decision rule. Having $CS_{agg} = \alpha_{agg_1} + \cdots + \alpha_{agg_n}$ consolidates us in this choice since if the only information we have is $(P_{agg_1}, \alpha_{agg_1}), \cdots, (P_{agg_n}, \alpha_{agg_n})$ then we first compute $\alpha_{agg_1} + \cdots + \alpha_{agg_n}$. Then if the sum is above the threshold, we use the decision rule. The indecision is handled in the same way as in the previous item replacing CS_j by α_{agg_i}.

- lg is the legitimacy. It is equal to the total number of voters over the number of authorized people to vote.

Example 3 *Below are the judgments of the members of our funding board. Let* $t = .8$. *Since* $CS_{agg} = .86 \geq .8$ *we use PBP. We get* $R = 1$. *The legitimacy of this decision is equal to* $4/5 = .8$. *We can observe that the collective decision of the members of the funding board agree on the three criteria but with a very low confidence about the relevance of* (S).

	P	Q	S	CS	R
M_1	(1, .5)	(0, .5)	(?, 0)	1	0
M_2	(?, .4)	(1, .4)	(1, .1)	.9	?
M_3	X	X	X	X	X
M_4	(1, .3)	(1, .4)	(?, .1)	.8	?
M_5	(1, .4)	(1, .3)	(1, .05)	.75	1
collective decision	(1, .4)	(1, .4)	(1, .06)	.86	1

Table 4: Example of judgment aggregation with confidence scores.

Example 4 *Let us now consider individuals who have to make a collective decision using the rule* $(P \wedge Q) \leftrightarrow R$, *which they think is not appropriate, i.e.* $CS < t$. *Suppose that their judgments are as in Table 5 and that* $t = .5$.

	P	Q	CS	R
M_1	(1, .1)	(1, .2)	.3	0
M_2	(0, .1)	(1, .1)	.2	0
M_3	(1, .2)	(0, .2)	.4	0
M_4	(1, .2)	(1, .1)	.3	1
M_5	(0, .2)	(0, .2)	.4	0
collective decision	(1, .16)	(1, .16)	.32	0

Table 5: Example of judgment aggregation with low confidence scores.

Since all five members assign a very low confidence score to the decision rule, they express their judgments on the criteria in the rule, but their decision on the conclusion R takes into account what they believe are the missing attributes. In this situation, the group will conclude $R = 0$ and they will be able to provide only partial reasons in support of their decisions. The group cannot reach a decision by proposition-wise majority voting on the criteria, as P and Q do not exhaust the reasons for or against R. Notice that the above line of reasoning also illustrates why our framework is less sensitive to paradoxical outcomes.

6 Some properties

Since JA problem is strongly connected with social choice problems, it is important to show that the proposed extension is still intuitively meaningful. We focus in this section on the description of the behavior of our approach and show that in extreme cases we recover the classical JA.

In our approach, the threshold governs our decision on whether we use the decision rule or not when computing R_j and also R_{agg}. The lower t is fixed, the more the group members are forced to adopt the decision rule, and the collective decision will be driven by the judgments on the criteria. This is coherent with our proposal to use PBP when $CS \geq t$. Symmetrically, setting a high t exposes the collective decision to be driven by the individual judgments on the conclusion, i.e. possibly less criteria will be considered important in the decision process. Again this is coherent with our proposal to use CBP when $CS < t$. The proposed extension has a nice behavior since it reduces into classical JA procedure when basic hypothesis are considered. More precisely, suppose that all weights are equal and $\alpha_{1j} + \cdots + \alpha_{nj} = 1$ (for $j = 1, \cdots, k$) and there are neither neutral judgments nor abstainers. In such a case:

(i) the notion of legitimacy is no longer meaningful since there are no abstainers,

(ii) P_{agg_i} is the majority of $P_{i1}, \cdots, P_{ik}$ as it is the case of proposition-wise majority voting used in classical JA,

(iii) $CS_j = \alpha_{1j} + \cdots + \alpha_{nj} = 1$ means that the group member j fully agrees on the decision rule so R_j is computed following the decision rule as it is the case in classical JA procedure,

(iv) $CS_{agg} = 1$ since $CS_j = 1$, $j = 1, \cdots, k$. Indeed we recover the dilemma of classical JA procedure on whether to use PBP or CBP. The selection between PBP and CBP depends on whether CS_{agg} is (respectively) high or low. When CBP is used, the group cannot provide the reasons for its decision. This is intuitive in our approach as, when CS_{agg} is low, the group members deemed the rule not appropriate for making that decision. Hence, the only reasonable position they can take is on the final conclusion,

without providing reasons. When, on the other hand, CS_{agg} is high (maybe even $CS_{agg} = 1$ as in standard JA), our procedure turns to the PBP. This is justified by the works on the epistemic perspectives of PBP and CBP [3, 9], whose main outcome is that PBP is better at tracking the right decision for the right reasons.

In addition, our aggregation procedure verifies some desirable properties such as *anonimity*, *no dictatorship* and *universal domain*. *Anonimity* and *no dictatorship* are the same as in standard JA. The first property requires that all group members who participated to the decision process (i.e. excluding abstainers) have equal weight in the aggregation. The absence of a dictator guarantees that there exists no single individual that always determines the collective decision. *Universal domain* guarantees that our aggregation rule takes the set of all *admissible* individual judgment sets and assigns a collective judgment set. In our framework, a judgment set is admissible if the weights of P_i, the judgments on the criteria P_i and on the conclusion R are assigned accordingly to the decision rule, the CS in the rule, and the individual judgment status. Clearly, the controversial *IIA* condition is not satisfied by our aggregation procedure. Depending on whether $CS \geq t$ or $CS < t$, our aggregation turns to PBP or CBP. Not satisfying the IIA condition, our approach provides an escape from the impossibility results plaguing standard JA.

7 Conclusion and Future Work

We extended classical JA procedure in order to offer a more realistic framework, and to escape the impossibility results. We introduce two main changes. Firstly, we define judgment status where a member can abstain or give binary or neutral judgments. Secondly, members assign weights to the criteria. A confidence score is computed on the basis of these weights. It expresses how well-suited a group member thinks that the adopted rule is for the decision process. This new representation of criteria allows us to avoid most cases of indecision by using specific decision rule (CBP or PBP) according to the value of the confidence score. Lastly, we introduce the notion of legitimacy that expresses to what extent the decision process is reliable. This work can be extended in different directions: *(i)* refine the notion of abstention by allowing abstention at the level of a criterion and study its impact on the legitimacy of the decision. This is appropriate when the weight associated to a criterion is equal to 0, *(ii)* extend our framework in order to treat extreme cases of indecision, *(iii)* investigate the relationship between criteria having the highest weight in our framework and works on coalitions [19]. We intend to study how group members can form coalitions and manipulate their confidence scores in order to drive the decision process in a particular direction. *(iv)* Lastly, investigate the relationship with opinion aggregation in order to go beyond binary judgments [20].

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